Physical Parameters Impact on the Ultrasound Low Frequency Reflected Signal from Rigid Porous Materials - Frequency Equivalent Fluid Theory Approach

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Abstract- The aim of this work is to study the effect of some physical parameters describing a rigid porous material on the reflected signal in the low frequency regime of ultrasound. The Johnson-Allard model modified by Sadouki [Phys. Fluids 33, (2021)] is considered in the context of the equivalent fluid theory that is a particular case of the Biot theory. A reflection coefficient expression is established in the frequency regime, which depends on four parameters, namely porosity, tortuosity, viscous characteristic length and thermal characteristic length as well as the two new viscous and thermal shape factors recently introduced in literature. The reflected signal is obtained by the product of the reflection coefficient and the incident field. A numerical simulation of the effect of each parameter on the reflected signal is studied and examined in the frequency domain.

Keywords – Porous materials, Equivalent fluid theory, Low-frequency of ultrasound, Reflected signal,

I. INTRODUCTION

Porous materials are ubiquitous in our daily lives and play an important role in our environment. A porous material consists of a solid phase saturated by a fluid phase. There are several types of porous materials. Such as glass wool which is used in buildings, household appliances, exterior surfaces and of course in the automotive industry to improve comfort and durability of vehicles. In the field of geophysics, we are also interested in the propagation of acoustic waves in porous rocks, to obtain information on the composition of soils and their fluid content. In medicine, the characterization of porous media such as trabecular bone is useful for the diagnosis of osteoporosis, a bone tissue disease that manifests itself by the deterioration of bone micro-architecture. Several physical and mechanical parameters characterize the porous medium. Some of these parameters have a significant effect on the amplitude of the transmitted and reflected wave, and others have little or no effect.

Previous studies [1-3] have been conducted to show the influence of the physical parameters describing the porous medium on the transmitted signal in the low frequency ultrasonic regime. In this section, a complementary study of the sensitivity of these same parameters involved in the Johnson-Allard model [4-6] and corrected by Sadouki [1] to classify them according to their order of influence on the reflected waves through a rigid porous medium in the low frequency ultrasonic regime. The parameters involved in the corrected model are porosity, tortuosity, viscous and thermal characteristic lengths as well as the two shape factors related to the thickness of the viscous boundary layer in the low frequency ultrasonic domain..

II.MODEL

In the case of an air saturated porous material, the structure remains immobile and undeformable with respect to the acoustic excitation; this is due to the heaviness and rigidity of the skeleton of the structure with respect to the air. We speak then of porous material with rigid structure, in this case, we use the equivalent fluid model [1-6] which is a special case of the Biot theory [7-9]. The fluid-structure interactions responsible for the attenuation of sound (particularly important in porous materials), give various physical properties not usual to the porous medium. Moreover, in the case of sound propagation, the frequency of motion plays an important role. At the low and high frequency ends, the equations governing the acoustic behavior of the fluid simplify and the parameters involved are different. The fluid-structure interactions, in the case of equivalent fluid theory, are taken into account by two frequency response factors: the dynamic tortuosity of the medium α(ω) given by Johnson et al [4,5] and the dynamic compressibility of the air in the porous material β(ω) given by Allard [6]. In the frequency domain, these factors are multiplied by the density and compressibility of the fluid. The high frequency ranges [4-6] are defined when the viscous and thermal skin depth $\delta(\omega) = \sqrt{\frac{2\eta}{\rho_0\omega}}$ and $\delta'(\omega) = \sqrt{\frac{2\eta}{P_r\rho_0\omega}}$ are smaller compared to the radius of the pores r (ρ_0 is the density of the saturating fluid, η is the viscosity, ω the pulsation frequency and $P_r$the Prandtl number). In the low frequency of the ultrasonic domain, α(ω) and β(ω) are given by [1] :

$$\alpha(\omega) = \alpha_\infty\left(1 + \frac{\delta(\omega)}{\Lambda}\left(\frac{2}{j}\right)^{\frac{1}{2}} + \xi\left(\frac{\delta(\omega)}{\Lambda}\right)^2\left(\frac{2}{j}\right) + \cdots\right) \qquad (1)$$

$$\beta(\omega) = 1 + (\gamma - 1)\left(\frac{\delta'(\omega)}{\Lambda'}\left(\frac{2}{j}\right)^{1/2} + (\xi' - 1)\left(\frac{\delta'(\omega)}{\Lambda'}\right)^2\left(\frac{2}{j}\right) + \cdots\right) \qquad (2)$$

where, $j = \sqrt{-1}$ and γ is the adiabatic constant.

The relevant physical parameters of the models are the high frequency limit of the tortuosity of the medium α_∞ initially introduced by Zwikker and Kosten [10], the viscous and thermalcharacteristic lengths Λ and Λ' introduced by Johnson and Allard [2-6]. The dimensionless parameter ξ introduced by Sadouki [1] is a shape factor related to the viscous skin depth correction of the air layer close to the tube surface where the velocity distribution is considerably disturbed by the viscous forces generated by the motionless frame in the ultrasonic low-frequency regime, and ξ' is the associated thermal counterpart.

Consider a homogeneous porous material that occupies the region $0 \le x \le L$. A sound pulse impinges normally on the medium. It generates an acoustic pressure field $p(x,)$ and an acoustic velocity field $v(x,\omega)$ within the material. The acoustic fields satisfy the Euler equation and the constitutive equation (along the x axis):

$$\rho_0\alpha(\omega)j\omega\, v(x,\omega) = \frac{\partial p(x,\omega)}{\partial x}, \qquad \frac{\beta(\omega)}{K_a}j\omega\, p(x,\omega) = \frac{\partial v(x,\omega)}{\partial x} \qquad (3)$$

With K_a is the compressibility modulus of the fluid.

Using the conditions continuity of the pressure and velocity fields at the boundary of the medium, we obtain the reflection coefficient of porous material given by [1]:

$$R = \frac{\left(1-\tilde{Z}^2\right)\sinh(j\tilde{k}L)}{2\tilde{Z}cosh(j\tilde{k}L)+\left(1+\tilde{Z}^2\right)\sinh(j\tilde{k}L)} \qquad (4)$$

Where $\tilde{Z} = \frac{1}{\phi}\sqrt{\frac{\alpha(\omega)}{\beta(\omega)}}$ is the normalized characteristic impedance of the material, ϕ is the porosity, and $\tilde{k} = \omega\sqrt{\frac{\rho_0\alpha(\omega)\beta(\omega)}{K_a}}$ is the wave number of the acoustic wave in the porous medium

The incident and reflected fieldsp^i and p^r_{sim} are related in the frequency domain by the reflection coefficient R:

$$p^r_{sim}(x,\omega) = R\, p^i(x,\omega) \qquad (5)$$

In the time domain, the reflected signal$p^r_{sim}(x,t)$ is obtained by taking the inverse Fourier transform of Eq. (5):

$$P^t(x,t) = \mathcal{F}^{-1}\left(R\, P^i(x,\omega)\right) \qquad (6)$$

III.SIMULATION STUDY

Figure 1 shows the simulated incident and reflected signal of a monolayer porous medium constructed in frequency via expression (5) and in time via equation (6) whose characteristics are: L = 2.0 cm, ϕ = 0.84, α_∞ = 1.2, Λ = 230 µm, Λ'/Λ = 2.0, ξ = 10.0 and ξ/ξ' = 3.0, These signals are constructed with central frequency peak at: 60 kHz. We notice, in the temporal representation, the two successive reflections of the acoustic wave on the first interface and the second interface of the porous medium.

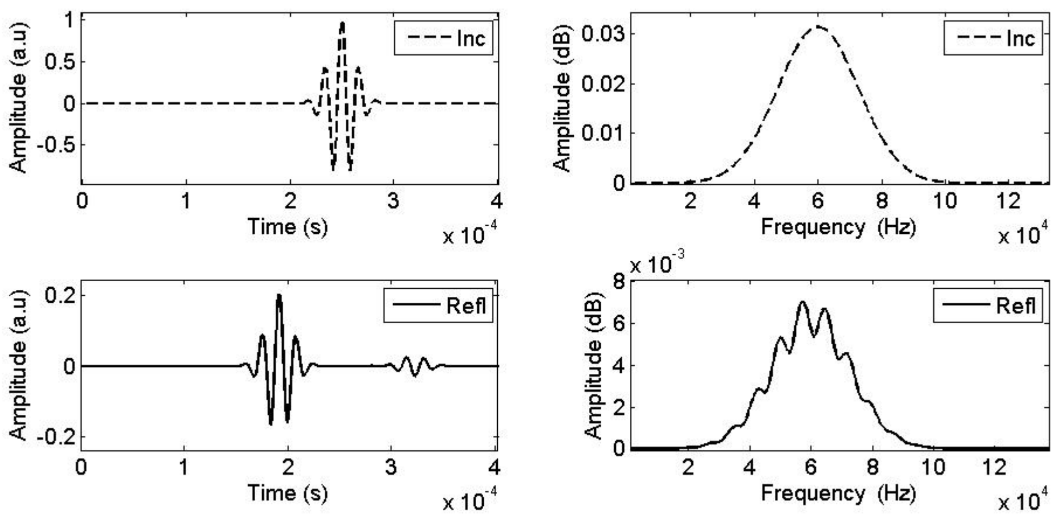

Figure 1. The incident and reflected signals of a monolayer porous medium constructed in frequency via expression (5) and in time via Eq. (6).

To study the influence of physical parameters, including porosity, tortuosity, viscous and thermal characteristic length and new shape factors on the reflected waves, we observe the effect of the variation of one parameter, the others remaining fixed, on the reflected signal in frequency domain given by expression (5).

A. Effect of porosity φ on the reflected signal

Figure (2) shows the effect of varying the porosity ϕ on the amplitude of the reflected signal, with the other parameters kept fixed, which are: $\alpha_\infty = 1.2$ $\Lambda = 230$ µm, $\Lambda'/\Lambda = 2$. $\xi = 10$. and $\xi/\xi' = 3$. The porosity ϕ varies from +30% to -30% of its initial value ($\phi 1 = 0.84$). Table 1 shows the ratio of variation of the reflection coefficient according to a variation of ±30% of each parameter.

Indeed, as shown in Table 1, for a frequency of 60 kHz, we observe a very great influence of the porosity on the reflected signal; when the porosity increases by +30%, the modulus of the reflected signal decreases by - 63.05% and for a variation of -30% the amplitude of the reflected signal grows by +78.47% %. On the other hand, and according to Table 1, we can see that the sensitivity of the porosity ϕ increases slightly with frequency.

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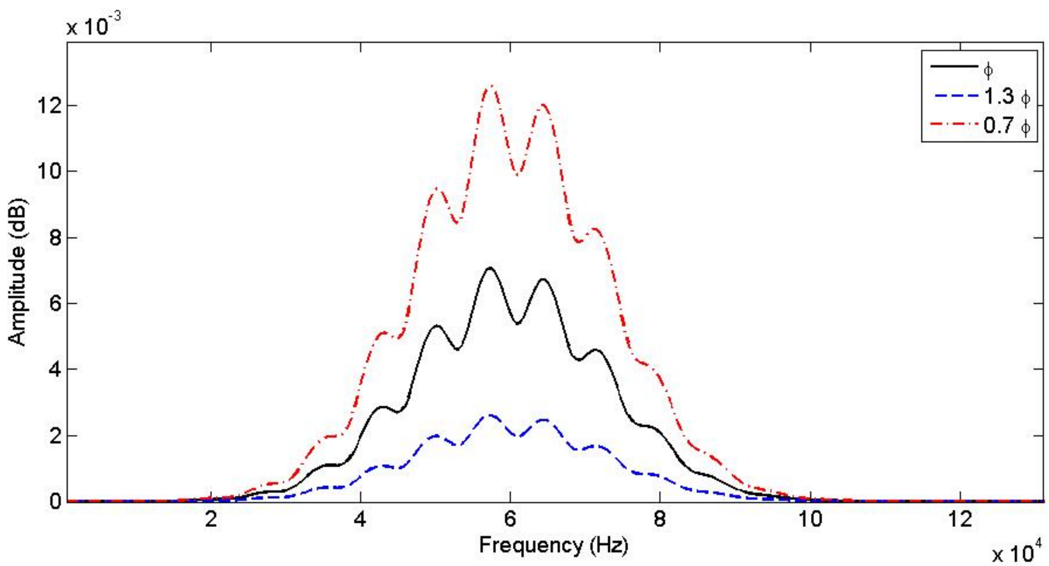

Figure 2 - The sensitivity of the porosity ϕ on the reflected signal at the 60 kHz frequency.

B. Effect of tortuosity α_∞ on the reflected signal

The sensitivity of the tortuosity α_∞ on the reflected signal is presented in Figure 3 for an excitation frequency of 60 kHz . For a variation of +30% of the initial value of tortuosity, the amplitude of the reflected signal increases by +25.65% and for a variation of -30%, the amplitude of the reflected signal decreases by -38.78%. We can confirm that the influence of the tortuosity is important on the reflected signal and that this influence increases with the frequency.

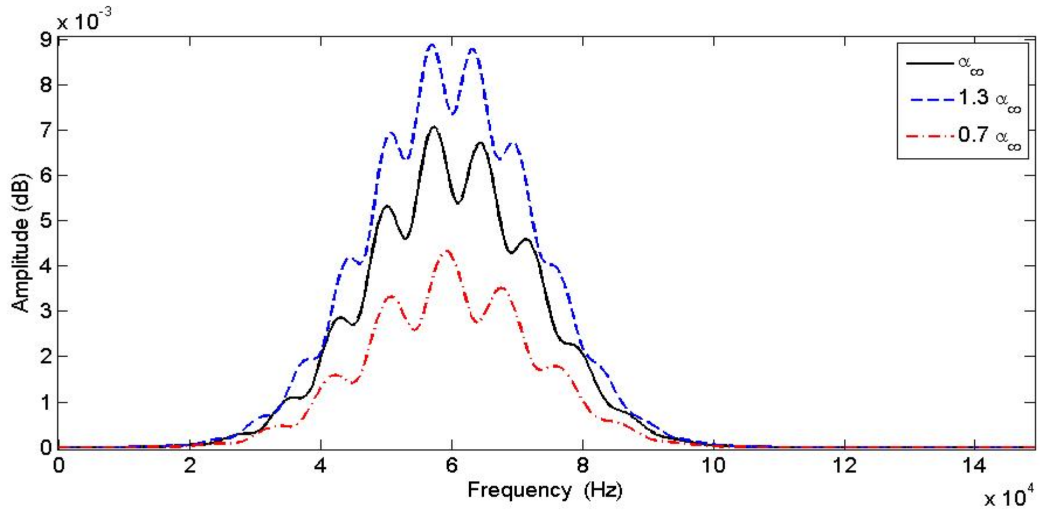

Figure 3. The sensitivity of the tortuosity on the transmitted signal

C. Effect of viscous characteristic length Λ on the reflected signal

The effect of the variation of the viscous characteristic length on the reflected signal at high frequency is shown in Figure 4. For an excitation frequency of 60 kHz, a +30% and -30% varying of Λ results in a +08.78% increase and -07.52% attenuation of the amplitude of the reflected signal, respectively (Table.1). On the other hand, the sensitivity decreases with frequency. We can conclude that the viscous characteristic length has an appreciable influence at high frequency on the reflected signal.

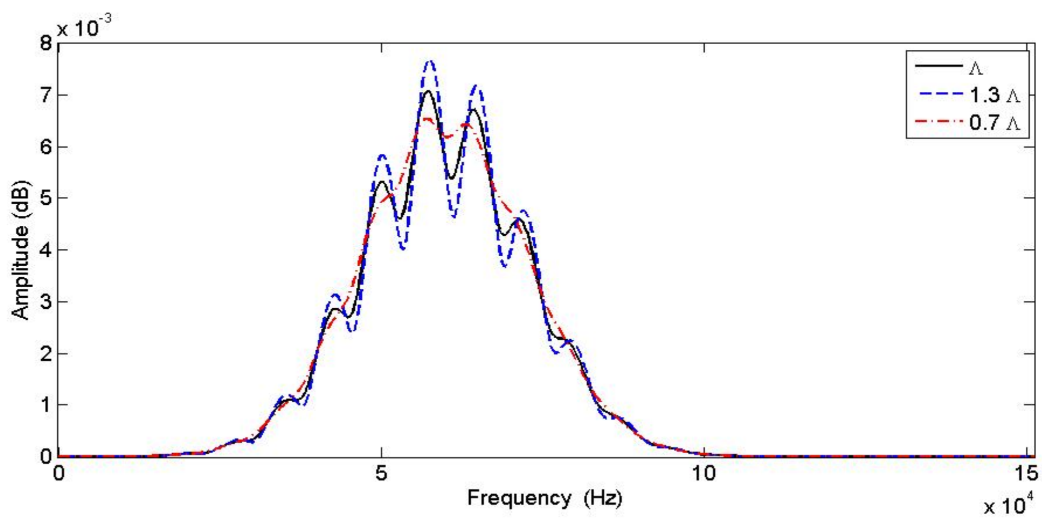

Figure 4. The sensitivity of the viscous characteristic length on the reflected signal at a frequency of 60 kHz.

*D. Effect of thermal characteristic length **Λ'** on the reflected signal*

The sensitivity of the thermal characteristic length Λ' on the reflected signal at low ultrasonic frequency is presented in Figure 5 for a variation from +30% to -30% of its initial value. According to Figure 5, for a frequency of 60 kHz, a very small influence of the thermal characteristic length on the reflected signal is observed; for an increase of +30%, the modulus of the reflected signal increases by 1.22% and for a variation of -30% the amplitude of the reflected signal decreases by -2.13%. On the other hand, and according to Table 1, it can be seen that the sensitivity of the thermal characteristic length decreases slightly with frequency.

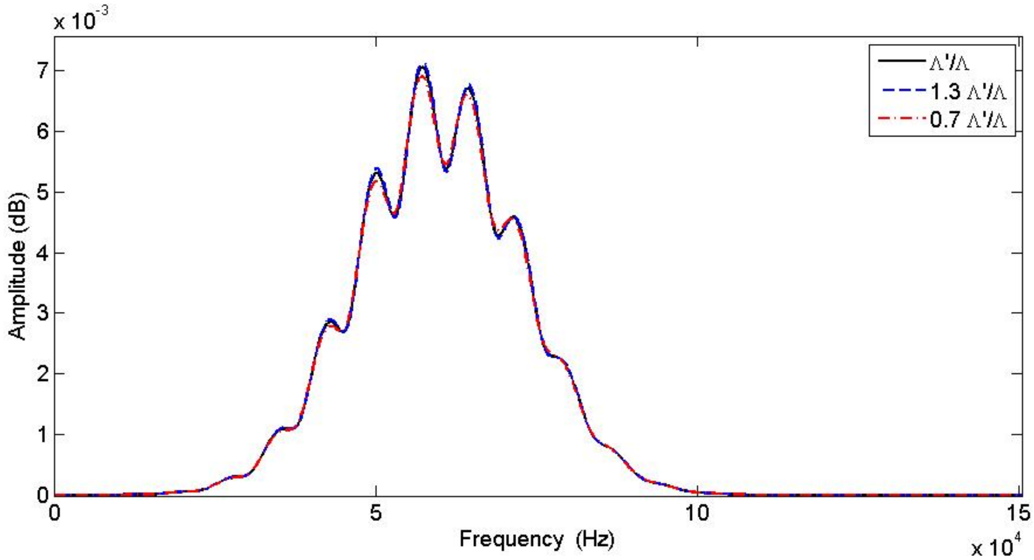

Figure 5. The sensitivity of the thermal characteristic length on the reflected signal for a frequency of 60 kHz.

*E. Effect of viscous shape factor **ξ** on the reflected signal*

The sensitivity of the shape factor ξ on the reflected signal in the low-frequency ultrasound regime is presented in Figure 6. From this figure, for a +30% variation of ξ the amplitude of the reflected signal regresses by -2.38% and for a -30% variation of ξ a +3.03% growth of the amplitude of the reflected wave is noticed. It is oncluded that the shape factor ξ also has a small influence on the reflected signal in the low frequency range of ultrasound and that this variation decreases as the frequency increases.

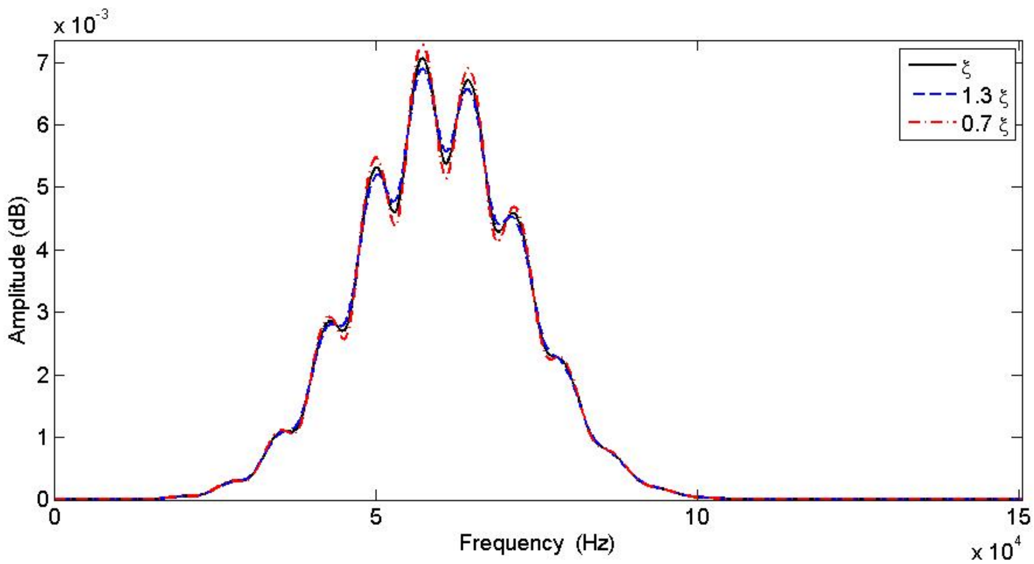

Figure 6. The sensitivity of the viscous shape factor ξ on the reflected signal for a frequency of 60 kHz.

F. Effect of thermal shape factor ξ' on the reflected signal:

The impact of varying the thermal shape factor ξ' on the high-frequency reflected signal is shown in Figure 7. For an excitation frequency of 60 kHz, a +30% and -30% variation of ξ/ξ' results in a +0.10% increase and -0.19% attenuation of the amplitude of the reflected signal, respectively. It can then be seen that this parameter has a very small dependence on the reflected signal in the low-frequency regime of ultrasound.

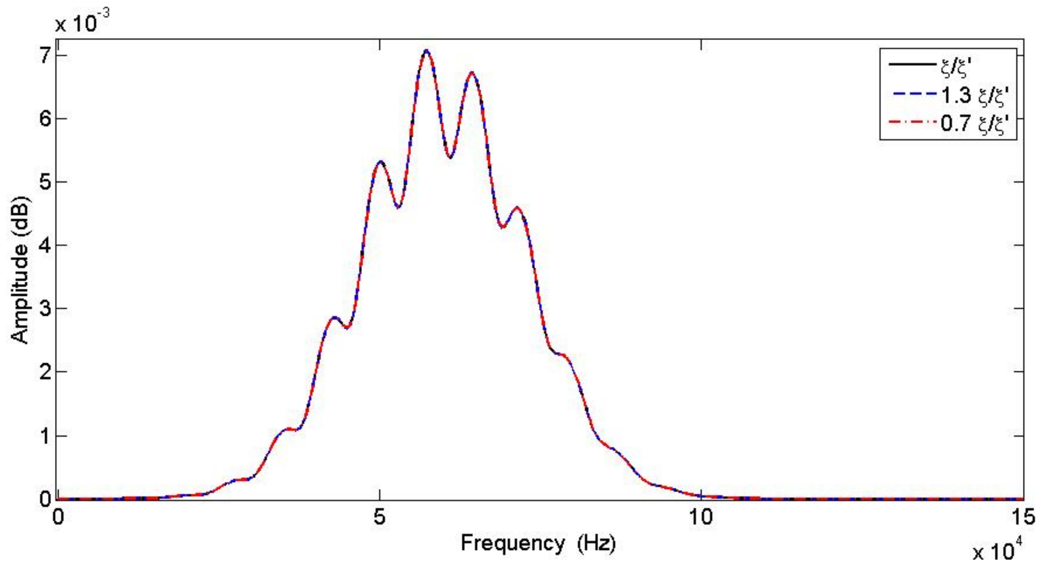

Figure 7. The sensitivity of the thermal shape factor ξ' on the reflected signal for a frequency of 60 kHz.

Table 1 summarizes the influence of porosity, tortuosity, viscous and thermal characteristic lengths as well as the two shape factors on the reflected signal in the low frequency regime of ultrasound (60-100) kHz.

Table 1 - Relative variation of the reflection coefficient $\frac{\Delta R}{R}$% corresponding to a variation of ± 30% of each physical parameter.

		$\frac{\Delta R}{R}\%$	
Parameters	Variations	80 kHz	100 kHz
Porosity	+30%	-63.05	-63.89
ϕ	-30%	+78.47	+79.82
Tortuosity	+30%	+25.65	+27.63
α_∞	-30%	-38.78	-41.08
Viscous characteristic length	+30%	+08.78	+07.88
Λ (μm)	-30%	-07.52	-06.27
Ratio of thermal and viscous characteristic lengths (Λ'/Λ)	+30%	+01.22	+00.98
	-30%	-02.13	-01.81
Viscous shape factor	+30%	-02.38	-01.96
ξ	-30%	+03.03	+02.48
Ration of viscous and thermal shape factors	+30%	+00.10	+00.15
ξ/ξ'	-30%	-00.19	-00.80

IV.CONCLUSION

A sensitivity study of the physical parameters describing the monolayer porous medium at low ultrasonic frequencies has been presented. The current study shows the influence of a ± 30% variation of these parameters, namely; porosity ϕ, tortuosity $\alpha\infty$, viscous and thermal characteristic length Λ and Λ' as well as the two viscous and thermal shape factors ξ and ξ' on the amplitude of the reflected signal. The sensitivity study was presented in the frequency domain. Following this study, it was found that porosity and tortuosity have a great influence on the reflected signal in the low ultrasonic frequency regime as well as tortuosity and viscous shape factor, while viscous characteristic length and shape factor have a small influence. The thermal parameters, namely thermal characteristic length and thermal shape factor, have a very small and negligible influence on the reflected signal. This sensitivity study greatly facilitates the inversion process to determine these parameters,

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