International Journal of Latest Trends in Engineering and Technology
Vol.(19)Issue(2), pp.028-035
DOI: http://dx.doi.org/10.21172/1.192.04
e-ISSN:2278-621X

Mathematical Modeling of Viscothermoelastic Diffusive Body with Phase Lags

Priyanka Kaushal
Department of Applied Sciences,
Chandigarh Group of Colleges, Landran, Punjab, India.

Harpal Singh
Department of Electronics & Communications Engineering,
Chandigarh Group of Colleges, Landran, Punjab, India.

Abstract - This investigation is focused on an axisymmetric problem of thick circular plate in a viscothermoelastic diffusive body within the context of dual-phase-lag heat transfer (DPLT) and dual-phase-lag diffusion (DPLD) models. The upper and lower surfaces of the thick plate are traction free and are subjected to particular types of thermal and chemical potential sources to depict the utility of the solution obtained. The solution is found by using Laplace and Hankel transform techniques and a direct approach without the use of potential functions. The expressions of displacement components, stresses and chemical potential are obtained in the transformed domain. Numerical inversion techniques are applied to obtain the resulting quantities in the physical domain. Some particular cases of results are also deduced.

I. INTRODUCTION

Diffusion is referred as the movement of particles from the regions of higher concentration to the regions of lower concentration. When temperature field, mass diffusion and strain are coupled together, Thermoelastic Diffusion (TD) takes place and it is a significant area among the oil companies for extraction of oil from the oil depositaries. The coupled thermo diffusive theory was formulated [1-3].

The dual phase lag (DPL) model was developed by taking into account the macroscopic scale and microscopic level. The microscopic scale depicts the interactions between electrons and phonons whereas macroscopic level observes retarding sources causing a delayed response. In this model, the Fourier law has been replaced by an approximation to the modified Fourier law with two distinct time translations: phase lag of temperature gradient τ_θ and phase lag of the heat flux τ_q[4-7].

Under the environmental factors like constant temperature, the linear theory of viscoelasticity plays a significant role. Due to the mechanical behavior of viscoelastic materials, the linear theory of viscoelasticity shows variations for the environmental factors such as humidity, presence of diffusion and temperature. The theory of bone viscoelasticity was presented and some theorems were studied [8]. Linear theories of viscoelasticity and Thermo –Viscoelasticity (TV) of binary mixtures were developed [9]. These theories are based on Kelvin–Voight viscoelastic model. The importance of viscoelastic materials in many branches of engineering and technology was discussed [10].

II. WORK DONE

Theory of TD was extended [11] for L-S (1967) model. In the thermoelastic theory with one relaxation time, the plane deformation because of thermal source was investigated [12]. A nonlinear theory of thermodynamics for elastic materials in which the particles have been subjected to classical displacement, temperature, and mass diffusion fields and whose microelements possess micro temperatures and micro concentrations was studied [13]. The authors analyzed thick circular plate problem including heat source in generalized TD [14]. A two-dimensional generalized TD problem due to laser pulse was analyzed [15]. TD with two time delays and Kernel functions was discussed [16]. The authors studied the homogeneous isotropic micropolar porous thermoelastic circular plate problem by using the Eigen value approach [17].

The effects of variable thermal conductivity and phase-lags with a cylindrical cavity were discussed in a TV solid [18]. The authors presented a new solution for nonlinear dual phase lagging heat conduction problem [19]. Under the gravitational field, the effect of rotation was studied for DPL model in a micropolar magnetothermoelastic solid [20].

A microporous thermoelastic circular plate considered in two dimensions was studied for the case of ramp type heating [21].

The authors have established expressions for the thermoelastic damping along with frequency shift of coupled DPL generalized visco-thermoelastic thin beam [22].Nanoscale heat transfer model was proposed. This model was based on the Caputo type fractional DPL heat conduction equation with the temperature jump boundary condition [23].

The explicit expression of basic solution of the system of differential equations in linear theory of micropolar viscoelasticity in terms of elementary function was formulated [24]. The authors discussed the reflection of waves in a micropolar thermo viscoelastic medium with microtemperature and study the effects of viscosity and microtemperature on the deviation of reflected waves [25]. The investigation was done for studying the disturbances in a homogeneous transversely isotropic magneto-visco thermoelastic rotating medium with two temperature due to thermomechanical sources [26]. The formulation of the generalized governing equations for the non-local heat conduction model was done and further the investigated a one-dimensional elastic half-space problem [27]. In order to investigate investigate the transient phenomena due to the influence of the magnetic field, the study based on Lord–Shulman (LS) theory of generalized thermo-visco-elasticity was done [28]. In the present investigation, two diffusion phase lags are used to introduce a generalized form of the equation of mass diffusion. One phase lag of diffusing mass flux vector is used to represent the detained time needed for diffusion of mass flux. Second phase lag of chemical potential is used to represent the detained time needed for inception of potential gradient.The fundamental equations for the isotropic VT diffusion medium in the light of DPL heat transfer and DPL diffusion models are presented.Two dimensional axisymmetric problem of thick circular plate due to thermal and chemical potential source is investigated.The components of displacement, stress, chemical potential, temperature and mass concentration are obtained by using Laplace Transform (LT) and Hankel Transform (HT) techniques and a direct approach without the use of potential functions. Some particular cases in this thread are also discussed.

III. BASIC EQUATIONS

For a homogeneous isotropic thermoelastic solid with DPL heat transfer and DPL diffusion models, the equations of motion, heat conduction and mass diffusion in the absence of body forces, heat sources and mass diffusion sources [29] are

$$(\lambda^* + \mu^*)\,\nabla\,(\nabla.\boldsymbol{u}) + \mu^*\Delta\boldsymbol{u} - \beta_1\nabla T - \beta_2\nabla C = \rho\ddot{\boldsymbol{u}}, \tag{1}$$

$$\left(1 + \tau_t\frac{\partial}{\partial t}\right)k^*\,T_{,ii} = \left(1 + \tau_q\frac{\partial}{\partial t} + {\tau_q}^2\frac{\partial^2}{\partial t^2}\right)\left[\rho C_E\dot{T} + \beta_1 T_0\dot{e}_{kk} + aT_0\dot{C}\right], \tag{2}$$

$$\left(1 + \tau_p\frac{\partial}{\partial t}\right)(D\beta_2\Delta(\nabla.\boldsymbol{u}) + Da\Delta T - Db\Delta C) + \frac{\partial}{\partial t}\left(1 + \tau_\eta\frac{\partial}{\partial t} + {\tau_\eta}^2\frac{\partial^2}{\partial t^2}\right)C = 0, \tag{3}$$

and the constitutive relations are

$$\sigma_{ij} = 2\mu^* e_{ij} + \delta_{ij}(\lambda^* e_{kk} - \beta_1 T - \beta_2 C), \tag{4}$$

$$\rho T_0 S = \left(1 + \tau_q\frac{\partial}{\partial t} + {\tau_q}^2\frac{\partial^2}{\partial t^2}\right)(\rho C_E T + \beta_1 T_0 e_{kk} + aT_0 C), \tag{5}$$

$$P = -\beta_2 e_{kk} - aT - bC, \tag{6}$$

$$\lambda^* = \lambda + \lambda^v\frac{\partial}{\partial t}, \tag{7}$$

$$\mu^* = \mu + \mu^v\frac{\partial}{\partial t} \tag{8}$$

In equations (1) – (8)

$\boldsymbol{u}$ - displacement vector, P - chemical potential per unit mass, C - mass concentration, Δ - Laplacian operator, ∇ - Nabla (Gradient) operator; λ , μ - Lame's constants, D - diffusivity, $\beta_1 = (3\lambda^* + 2\mu^*)\alpha_t$, $\beta_2 = (3\lambda^* + 2\mu^*)\alpha_c$, α_c – is linear diffusion expansion Coefficient, α_t – is thermal linear expansion Coefficient, τ_t – temperature gradient phase lag, τ_q - heat flux phase lag, τ_p - chemical potential phase lag, τ_η - diffusing mass flux vector phase lag, k^* - Thermal conductivity, a - coefficient describing the measure of thermodiffusion, b - coefficient describing the measure of mass diffusion effect, C_E - specific heat. In these equations, superposed dot represents a derivative w.r.t. time and comma followed by suffix represents the spatial derivative.

IV. METHODOLOGY

We assume a thick circular plate whose thickness is 2b and occupying the space D defined by $0 \leq r \leq \infty$, $-b \leq z \leq b$ in viscothermoelastic diffusion with dual phase lag model. We take the cylindrical polar co-ordinate system (r, θ, z) whose symmetry is about z-axis.

The plate is assumed to subject to chemical potential source and an axisymmetric heat supply whose boundary is stressed free and depends upon the axial and radial directions of cylindrical polar co-ordinate system. T_0 is the initial temperature, considered to be constant in the thick circular plate. The heat flux and chemical potential source are taken along diminishing of the stress components on upper and lower boundary surfaces with traction free boundary $z = \pm b$ and these are of unit magnitude. In these circumstances, it is required to determine the physical quantities in a thick circular plate. Since it is a plane axisymmetric problem, the field component $u_\theta = 0$, and u_r, u_z, T and C are independent of θ and we will confine the analysis to a two dimensional problem with

$$\boldsymbol{u} = (u_r, 0, u_z) \tag{9}$$

Equations (1) - (6) with aid of equation (9) gives

$$\frac{\partial e}{\partial r} + \frac{\mu}{(\lambda+\mu)}\left(\Delta - \frac{1}{r^2}\right)u_r - \frac{\beta_1}{(\lambda+\mu)}\frac{\partial T}{\partial r} - \frac{\beta_2}{(\lambda+\mu)}\frac{\partial C}{\partial r} = \frac{\rho}{(\lambda+\mu)}\frac{\partial^2 u_r}{\partial t^2}, \tag{10}$$

$$\frac{\partial e}{\partial z} + \frac{\mu}{(\lambda+\mu)}\Delta u_z - \frac{\beta_1}{(\lambda+\mu)}\frac{\partial T}{\partial z} - \frac{\beta_2}{(\lambda+\mu)}\frac{\partial C}{\partial z} = \frac{\rho}{(\lambda+\mu)}\frac{\partial^2 u_z}{\partial t^2}, \tag{11}$$

$$(1+\tau_t\frac{\partial}{\partial t})\Delta T = \left(1 + \tau_q\frac{\partial}{\partial t} + \frac{\tau_q{}^2}{2}\frac{\partial^2}{\partial t^2}\right)\left[\frac{\rho C_E T}{k^*} + \frac{\beta_1 T_0\frac{\partial e}{\partial t}}{k^*} + \frac{a T_0\frac{\partial C}{\partial t}}{k^*}\right], \tag{12}$$

$$(1+\tau_p\frac{\partial}{\partial t})(D\beta_2\Delta e + Da\Delta T - Db\Delta C) + \frac{\partial}{\partial t}\left(1 + \tau_\eta\frac{\partial}{\partial t} + \frac{\tau_\eta{}^2}{2}\frac{\partial^2}{\partial t^2}\right)C = 0, \tag{13}$$

Components of stress & chemical potential given by equations (14) - (20) reduce to the form

$$\sigma_{rr} = 2\mu e_{rr} + \lambda e - \beta_1 T - \beta_2 C, \tag{14}$$

$$\sigma_{\theta\theta} = 2\mu e_{\theta\theta} + \lambda e - \beta_1 T - \beta_2 , \tag{15}$$

$$\sigma_{zz} = 2\mu e_{zz} + \lambda e - \beta_1 T - \beta_2 C , \tag{16}$$

$$\sigma_{rz} = \mu e_{rz} , \tag{17}$$

$$\sigma_{r\theta} = 0, \tag{18}$$

$$\sigma_{z\theta} = 0 , \tag{19}$$

$$P = -\beta_2 e - aT + bC , \tag{20}$$

where

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}, e_{rr} = \frac{\partial u_r}{\partial r}, \quad e_{\theta\theta} = \frac{u_r}{r}, e_{zz} = \frac{\partial u_z}{\partial z}, e_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right),$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r} \quad . \tag{21}$$

To facilitate the solution, the following dimensionless quantities are introduced

$$r' = \frac{\omega_1^*}{c_1}r, z' = \frac{\omega_1^*}{c_1}z, (u_r', u_z') = \frac{\omega_1^*}{c_1}(u_r, u_z), t' = \omega_1^* t,$$

$$(\sigma_{rr}', \sigma_{\theta\theta}', \sigma_{zz}', \sigma_{rz}') = \frac{1}{\beta_1 T_0}(\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{rz}), \quad P' = \frac{P}{\beta_2} = -e - \frac{a\rho c_1^2}{\beta_1\beta_2}T + \frac{b\rho c_1^2}{\beta_2{}^2}C,$$

$$T' = \frac{\beta_1}{\rho c_1^2}T, C' = \frac{\beta_2}{\rho c_1^2}C, \quad (\tau_q', \tau_t', \tau_p', \tau_\eta') = \omega_1^*(\tau_q, \tau_t, \tau_p, \tau_\eta), \tag{22}$$

where

$$\omega_1^* = \frac{\rho C_E c_1^2}{K}, c_1^2 = \frac{\lambda^* + 2\mu^*}{\rho} \tag{23}$$

Using (22) and (23) in equations (10)-(13) and after that suppressing the primes and then applying the LT &HT defined by

$$\bar{f}(r,z,s) = \int_0^\infty f(r,z,t)e^{-st}dt \tag{24}$$

$$\hat{f}(\xi,z,s) = \int_0^\infty \bar{f}(r,z,s)rJ_n(r\xi)dr \tag{25}$$

Now applying on resulting equations and after simplification, we obtain

$$(\Delta - s^2)\bar{e} - \Delta\bar{T} - \Delta\bar{C} = 0, \tag{26}$$

$$(\Delta - \zeta_{21} s)\bar{T} - \zeta_{22} s\bar{C} - \zeta_{23} s\, \bar{e}=0, \tag{27}$$

$$\zeta_{31}\Delta\bar{T} - (\zeta_{32}\Delta - \zeta_{33} s)\bar{C} + \zeta_{34}\Delta\bar{e} = 0, \tag{28}$$

where

$$\zeta_{21} = \frac{\tau_q^1}{\tau_t^1} k\ , \quad \zeta_{22} = \frac{k^* a T_0 \tau_q^1}{\rho c_E \beta_2 \tau_t^1}, \quad \zeta_{23} = \frac{k^* {\beta_1}^2 T_0}{\rho^2 c_E c_1^2} \frac{\tau_q^1}{\tau_t'},$$

$$\zeta_{31} = \frac{D a \rho c_1^2}{\beta_1}\ , \quad \zeta_{32} = Db \frac{\rho c_1^2}{\beta_2}, \quad \zeta_{33} = \frac{\tau_\eta^1 k c_1^2}{\tau_p^1 \beta_2 c_E}, \quad \zeta_{34} = D\beta_2\ ,$$

$$\tau_q^1 = 1 + s\tau_q + \frac{s^2 {\tau_q}^2}{2}, \tau_\eta^1 = 1 + s\tau_\eta + \frac{s^2 {\tau_\eta}^2}{2}, \tau_p^1 = 1 + s\tau_p\ , \tau_t^1 = 1 + s\tau_t\ . \tag{29}$$

Eliminating the dilatation strain $\bar{e}$, the temperature $\bar{T}$ and concentration $\bar{C}$ from the equations (26) - (28), we obtain

$$(\Delta^3 - a_1\Delta^2 + a_2\Delta - a_3)\,(\bar{e}, \bar{T}, \bar{C}) = 0 \tag{30}$$

Applying the HT defined by (25) on (30) and after simplification, we obtain

$$\left(\frac{d^6}{dz^6} - a_1 \frac{d^4}{dz^4} + a_2 \frac{d^2}{dz^2} - a_3\right)\left(\hat{e}, \hat{T}, \hat{C}\right) = 0 \tag{31}$$

where

$$a_1 = \frac{\zeta_{22}\zeta_{33} + \zeta_{23}\zeta_{32} - \zeta_{33}\zeta_{21} + \zeta_{22}\zeta_{31} + s^2\zeta_{32} + \zeta_{21}\zeta_{32} + \zeta_{32}' + \zeta_{22}\zeta_{31}}{\zeta_{32} - \zeta_{33}}\ , \tag{32}$$

$$a_2 = \frac{\zeta_{32}' + s^2(\zeta_{21}\zeta_{32} + \zeta_{32}' + \zeta_{22}\zeta_{31}) + \zeta_{21}\zeta_{32}'}{\zeta_{32} - \zeta_{33}}\ , \tag{33}$$

$$a_3 = \frac{s^2 \zeta_{21}\zeta_{32}'}{\zeta_{32} - \zeta_{33}}\ . \tag{34}$$

General solution of equation (31) may be written as

$$\hat{T} = \hat{T}_1 + \hat{T}_2 + \hat{T}_3 \tag{35}$$

where

$\hat{T}_i$ (i= 1, 2, 3) is the general solution of the homogenous differential equation

$$(D^2 - \xi^2 - k_i^2)\hat{T}_i = 0, \quad (i = 1,2,3) \tag{36}$$

where

$D = \frac{d}{dz}$ and $k_i\,(i = 1,2,3)$ are the roots of the characteristic equation

$$(D^6 - a_1 D^4 + D^2 a_2 - a_3) = 0 \tag{37}$$

The solution of equation (36) is expressed as

$$\hat{T}_i = \sum_{i=1}^{3} A_i(\xi, s)\cosh\,(q_i z), \tag{38}$$

where

$$q_i^2 = \xi^2 + {k_i}^2, \quad (i = 1,2,3)$$

Similarly, the dilation $\hat{e}$ and concentration $\hat{C}$ are obtained from equations (26) - (28) as

$$\hat{e} = \sum_{i=1}^{3} f_i A_i(\xi, s)\cosh\,(q_i z), \tag{39}$$

$$\hat{C} = \sum_{i=1}^{3} d_i A_i(\xi, s)\cosh(q_i z)\ , \tag{40}$$

where d_i , f_i are the coupling constants.

$$d_i = \frac{\zeta_{34} q_i^4 - 6\zeta_{21}\zeta_{34} q_i^2 + \zeta_{23}\zeta_{31}}{-q_i^2(\zeta_{22}\zeta_{34} + \zeta_{23}\zeta_{32}) + \zeta_{23}\zeta_{33}}, \tag{41}$$

$$f_i = \frac{-\zeta_{32} q_i^4 + (\zeta_{33} + \zeta_{21}\zeta_{32}) q_i^2 - \zeta_{21}\zeta_{33}}{-q_i^2(\zeta_{22}\zeta_{34} + \zeta_{23}\zeta_{32}) + \zeta_{23}\zeta_{33}}, \quad (i = 1,2,3) \tag{42}$$

and A_i are arbitrary constants

Applying inversion of HT on equations (38)- (40), we obtain

$$\bar{T}_i = \int_0^\infty \{\textstyle\sum_{i=1}^{3} A_i(\xi, s)\cosh(q_i z)\}\xi J_0(\xi r)\,d\xi, \tag{43}$$

$$\bar{C} = \int_0^\infty \{\sum_{i=1}^3 d_i A_i(\xi, s) \cosh(q_i z)\} \xi J_0(\xi r) d\xi, \quad (44)$$

$$\bar{e} = \int_0^\infty \{\sum_{i=1}^3 f_i A_i(\xi, s) \cosh(q_i z)\} \xi J_0(\xi r) d\xi, \quad (45)$$

Making use of equations (26) - (28) and (43) - (45), we get displacement components in transformed domain as

$$\bar{u}_r(r, z, s) = \int_0^\infty \xi^2 J_1(\xi r) \left[E(\xi, s) \cosh(qz) + \sum_{i=1}^3 \frac{\lambda^0{}_i A_i}{\left(\frac{\mu q_i^2}{\rho c_1^2} - s^2\right)} \cosh(q_i z) \right] d\xi \quad (46)$$

$$\bar{u}_z(r, z, s) = \int_0^\infty \xi J_0(\xi r) \left[F(\xi, s) \sinh(qz) - \sum_{i=1}^3 \frac{\lambda^0{}_i A_i}{\left(\frac{\mu q_i^2}{\rho c_1^2} - s^2\right)} \sinh(q_i z) \right] d\xi \quad (47)$$

where

$$\lambda^0{}_i = \left(\frac{\lambda^* + \mu^*}{\rho c_1^2} f_i - 1 - d_i\right), (\xi, s) = \frac{\xi^2 E(\xi, s)}{q}, \; q = \sqrt{\xi^2 + \frac{\rho c_1^2}{\mu^*} s^2} \; (i = 1,2,3) \quad (48)$$

Applying the LT defined by equation (24) on equations (42) - (45) and (48) yields stress components & chemical potential in Laplace transformed domain and then substituting the values of $\bar{u}_r, \bar{u}_z, \bar{T}_i, \bar{C}$ from equation (43) - (44) and equations (46) - (48) in the resulting equations yield the stress component and chemical potential as

$$\bar{\sigma}_{\theta\theta} = \frac{2\mu^*}{\beta_1 T_0} \int_0^\infty \xi^2 J_1(\xi r) \left[E(\xi, s) \cosh(qz) + \sum_{i=1}^3 \frac{\lambda^0{}_i A_i}{\left(\frac{\mu q_i^2}{\rho c_1^2} - s^2\right)} \cosh(q_i z) \right] d\xi + \int_0^\infty \sum_{i=1}^3 \eta_i A_i \cosh(q_i z) \, \xi J_0(\xi r) \, d\xi, \quad (49)$$

$$\bar{\sigma}_{rr} = \frac{2\mu^*}{\beta_1 T_0} \int_0^\infty \xi^3 \left(\frac{1}{\xi r} J_1(\xi r) - J_0(\xi r)\right) \left[E(\xi, s) \cosh(qz) + \sum_{i=1}^3 \frac{\lambda^0{}_i A_i}{\left(\frac{\mu q_i^2}{\rho c_1^2} - s^2\right)} \cosh(q_i z) \right] d\xi + \int_0^\infty \sum_{i=1}^3 \eta_i A_i \cosh(q_i z) \, \xi J_0(\xi r) \, d\xi, \quad (50)$$

$$\bar{\sigma}_{zz} = \frac{2\mu^*}{\beta_1 T_0} \int_0^\infty \xi J_0(\xi r) \left[F(\xi, s) q \cosh(qz) + \sum_{i=1}^3 \frac{\lambda^0{}_i A_i q_i^2}{\left(\frac{\mu q_i^2}{\rho c_1^2} - s^2\right)} \cosh(q_i z) \right] d\xi + \int_0^\infty \sum_{i=1}^3 \eta_i A_i \cosh(q_i z) \, \xi J_0(\xi r) \, d\xi, \quad (51)$$

$$\bar{\sigma}_{rz} = \frac{\mu^*}{2\beta_1 T_0} \int_0^\infty \xi^2 J_1(\xi r) \left[\left(\frac{q^2 - \xi^2}{q}\right) E(\xi, s) \, q\sinh(qz) + 2 \sum_{i=1}^3 \frac{\lambda^0{}_i A_i}{\left(\frac{\mu q_i^2}{\rho c_1^2} - s^2\right)} q_i \sinh(q_i z) \right] d\xi, \quad (52)$$

$$\bar{P}(r, z, s) = \int_0^\infty \sum_{i=1}^3 \zeta_i A_i \cosh(q_i z) \, \xi J_0(\xi r) \, d\xi, \quad (53)$$

where

$$\eta_i = \left(\frac{f_i - \rho c_1^2 - \rho c_1^2 d_i}{\beta_1 T_0}\right), F(\xi, s) = \frac{\xi^2 (E(\xi, s))}{q}, \; \zeta_i = \left(-\beta_2 f_i - \frac{\rho c_1^2}{\beta_1} + \frac{b \rho c_1^2}{\beta_2} d_i\right), \; (i = 1,2,3) \quad (54)$$

V. BOUNDARY CONDITIONS

We consider thermal source & chemical potential source with diminishing of the stress components on surface at $z = \pm b$. These may be written as mathematically

$$\frac{\partial T}{\partial z} = \pm g_0 F(r, z), \quad (55)$$

$$\sigma_{zz} = 0, \quad (56)$$

$\sigma_{rz} = \sigma_{rz} = 0,$ (57)

$P = \delta(t)H(a-r)$ (58)

where

$F(r,z) = z^2 e^{-\omega r},$ (59)

g_0 is constant temperature of boundary, $\delta(\ )$: the Dirac delta function and H( ) is Heaviside unit step function. Applying LT defined by equation (24) and HT defined by equation (25) the boundary conditions (55) - (58), yield

$$\frac{d\hat{T}}{dz} = g_0 \bar{F}(\xi, z) \quad (60)$$

$$\widehat{\sigma_{zz}} = 0 \quad (61)$$

$$\widehat{\sigma_{rz}} = 0 \quad (62)$$

$$\hat{P} = \frac{aJ_1(\xi a)}{\xi} \quad (63)$$

where

$$\hat{F}(\xi, z) = \frac{z^2\omega}{(\xi^2+\omega^2)^{3/2}} \quad (64)$$

Applying the inverse HT on equations(60) - (63) and then substituting the values of $\bar{T}$, $\overline{\sigma_{zz}}$, $\overline{\sigma_{rz}}$, $\bar{P}$ from equations (43), (51) - (53) in the resulting equations, we obtain the values of unknown parameters as

$$A_1 = \frac{\Delta_1}{\Delta}, A_2 = \frac{\Delta_2}{\Delta}, A_3 = \frac{\Delta_3}{\Delta}, E(\xi, s) = \frac{\Delta_4}{\Delta} \quad (65)$$

where

$$\Delta = \begin{vmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & 0 \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \frac{-2\mu}{\beta_1 T_0}\xi^2 q\cos(qb) \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \frac{q^2-\xi^2}{q}\sinh(qb) \\ \Delta_{41} & \Delta_{42} & \Delta_{43} & 0 \end{vmatrix}$$

$\Delta_{1i} = q_i \sinh(q_i b)$, $\Delta_{2i} = (\mu_i q_i^2 + \eta_i)\cosh(q_i b)$, $\Delta_{3i} = \mu_i q_i \sinh(q_i b)$, $\Delta_{4i} = \zeta_i \cosh(q_i b)$, ($i$=1,2,3)

and Δ_i is obtained from , by interchanging i^{th} column with $\left[g_0 F(\xi, b)\ \ 0\ \ 0\ \ \frac{aJ_1(\xi a)}{\xi}\right]^{t_r}$, where t_r denotes the transpose.

Components of displacement, stress, temperature change & chemical potential are obtained by substituting the values of A_i from equation (65)in the equations (46) - (47) & (49) - (53) for viscothermoelastic with DPL model.

VI. INVERSION OF TRANSFORM

To get solution of problem in physical domain, we need to invert transforms. The components of displacement, temperature change, normal stress, mass concentration, sheer stress and chemical potential are functions of z, the parameter of LT and HT s and ξ respectivly and thus are of the form $f(\xi, z, s)$. We first invert the HT which gives the LT expression $\bar{f}(r, z, s)$ of the function $f(r, z, t)$ as

$$\bar{f}(r, z, s) = \int_0^\infty \xi f(\xi, z, s) J_n(\xi r) d\xi \quad (66)$$

The inversion of transformed function $\bar{f}(r, z, s)$ is

$$\bar{f}(x_1, x_3, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\eta x_1} \hat{f}(\eta, x_3, s)\, d\eta = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\cos(\eta x_1) f_e - i\sin(\eta x_1) f_0|\, d\eta \quad (67)$$

VII. PARTICULAR CASES

(i) If Q_i= 0 (i=1,2,3) in equations (46) - (47) & (49) - (53) we acquire the corresponding expressions for thermoelastic with DPL model.

(ii) If we ignore the diffusion effect (i.e. $\beta_2, a, b = 0$) in equations ((46) - (47) & (49) - (53), we get expressions for the displacement components, stress, temperature change and chemical potential for viscothermoelastic isotropic half space.

(iii) If $\tau_p = \tau_\eta = 0$, in equations (46) - (47) & (49) - (53), then the corresponding relations reduces to viscothermoelastic with DPL model.

(iv) If $\tau_q = 0$ and $\tau_p = 0$, in equations (46) - (47) & (49) - (53), then corresponding results for DPL heat transfer and DPL diffusion models reduce to single phase lag heat model and single phase lag diffusion model for viscothermoelastic model.

VIII. CONCLUSION

The present investigation is focused on the behavior of thick circular plate in viscothermoelastic diffusion with and without DPL due to thermal and chemical potential sources. The fundamental equations for the isotropic TD medium in the light of DPL heat transfer and DPL diffusion models in axisymmetric form are presented. LT and HT are applied to get the solution of problem. Present problem gives results of great use for two dimensional dynamic responses due to many sources in viscothermoelastic diffusion which has many industrial and geophysical applications. A sound impact of viscosity on components of stress, chemical potential and displacement in the thick circular plate has been observed.

REFERENCES

[1] Nowacki, W., (1974a), "Dynamic problems of thermodiffusion in solids", Bull. Acad. Polon.Sci. Serie Sci. Techn., 22(1), pp 43-55.

[2] Nowacki, W., (1974b) , ibid. II, 22(3) , pp 129-205.

[3] Nowacki, W., (1974c), ibid. III, 22(4) , pp 161-257.

[4] Tzou, D. Y., (1995a), "A unified approach for heat conduction from macro-to-micro-scales", J. Heat Transfer, 117, pp 8-16.

[5] Tzou, D. Y., (1995b), "The generalized lagging response in small scale and high rate heating", International journal of Heat Transfer, 38(17), pp 3231-3240.

[6] Tzou, D. Y., (1996), "Macro-to-micro scale heat transfer: the lagging behavior", Taylor & Francis, Washington, DC, CRC Press.

[7] Tzou, D. Y., (2014), "Macro- to Microscale Heat Transfer: The Lagging Behavior", 2nd edition, Wiley, USA.

[8] Quintanilla, R., (2005), "Existence and exponential decay in the linear theory of viscoelastic mixtures", European Journal of Mechanics A/Solids, 24, pp 311–324.

[9] Iesan, D. and Nappa, L., (2008), "On the theory of viscoelastic mixtures and stability", Mathematics and Mechanics of Solids, 13, pp 55–80.

[10] Lakes, R., (2009), "Viscoelastic Materials", Cambridge University Press, UK.

[11] Sherief, H.H., Hamza, F.A, and Saleh, H.A, (2004), "The theory of generalized thermoelastic diffusion", International journal of Engineering Science, Volume 42, pp 591- 608.

[12] Sherief, H.H and Saleh, H.A, (2005), "A half space problem in the theory of generalized thermoelastic diffusion", International journal of Solid and Structures, Volume 42, pp 4484 – 4493

[13] Aouadi, M., Ciarletta, M. and Tibullo, V., (2016), "A thermoelastic diffusion theory with microtemperatures and microconcentrations",Journal of Thermal Stresses, Volume 40, Issue 4, pp 486-501.

[14] Tripathi, J.J., Kedar, G. D. and Deshmukh, K.C., (2016), "Generalized thermoelastic diffusion in a thick circular plate including heat source", Alexandria Engineering Journal, Volume 55, Issue 3, pp 2241-2249.

[15] Abbas, I.A. and Marin, M., (2017), "Analytical Solutions of a Two-Dimensional Generalized Thermoelastic Diffusions Problem Due to Laser Pulse", Iranian Journal of Science and Technology, Transactions of Mechanical Engineering , Volume 42 issue 1, pp. 57-71

[16] El-Karamany, A.S., Ezzat, M.A. and · El-Bary, A.A., (2018), "Thermodiffusion with two time delays and Kernel functions", Mathematics and Mechanics of Solids, 23(2), pp 195–208.

[17] Kumar.R, Kaushal. P and Sharma. R, (2017); Axisymmetric Vibration for Micropolar Porous Thermoelastic Circular Plate, *Int. J. Of Applied Mechanics And Engineering, Vol.22, No.3, Pp.583-600.*

[18] Zenkour, A.M., (2016), "Effects Of Phase-Lags And Variable Thermal Conductivity in a Thermoviscoelastic Solid With A Cylindrical Cavity", Honam Mathematical J. 38, No. 3, pp 435–454.

[19] Noroozi., M.J.,Saedodin, S. and Ganji, D.D., (2016), "A new solution for nonlinear Dual Phase Lagging heat conduction problem", Alexandria Engineering Journal, Volume 55, Issue 2, pp 1745-1752.

[20] Othman, M.I.A. and Abd-Elaziz, E., (2017), "Effect of rotation on a micropolar magnetothermoelastic solid in dual-phase-lag model under the gravitational field", Microsystem Technologies, Issue 10.

[21] Kumar.R, Kaushal. P and Sharma. R, (2017) "Eigen value approach for dual phase lag micropolar porous thermoelastic circular plate with ramp type heating", Multidiscipline Modeling in Materials and Structures, Vol. 13 Issue: 4, pp.550-567.

[22] Grover, D. and Seth, R. K., (2018), "Viscothermoelastic micro-scale beam resonators based on dual-phase lagging model", Microsystem Technologies, Volume 24, Issue 3, pp 1667–1672.

[23] Cui-Ji, C., Dai., W. and Sun, Z., (2017), "Numerical Method for Solving the Time-Fractional Dual-Phase-Lagging Heat Conduction Equation with the Temperature-Jump Boundary Condition", Journal of Scientific Computing , pp 1–30.

[24] Svanadze, M.M.,(2017), "On the solutions in the linear theory of micropolar viscoelasticity", Mechanics Research Communications, Volume 81, pp 17-25.

[25] Abo-Dahab, S.M., Jahangir, A., Muhammad,N., Farwa, S., Bashir, Y. and Usman, M., (2018), "Propagation phenomena in a visco-thermo-micropolar elastic medium under the effect of micro-temperature", Results in Physics, Volume 8, pp 793-798.

[26] Kumar.R, Kaushal. P and Sharma. R(2018);Transversely Isotropic Magneto-Visco Thermoelastic Medium with Vacuum and without Energy Dissipation,Journal of Solid Mechanics Vol. 10, No. 2 pp. 416-434.

[27] Gupta. M and Mukhopadhyay. S (2019) A study on generalized thermoelasticity theory based on non-local heat conduction model with dual-phase-lag, Journal of Thermal Stresses, 42:9, 1123-1135.
[28] Mondal, S. Interactions of a Heat Source Moving over a Visco-Thermoelastic Rod kept in a Magnetic Field in the Lord–Shulman Model under a Memory Dependent Derivative. *Comput Math Model* **31,** 256–276 (2020).
[29] Kumar, R. and Gupta, V., (2014), "Plane wave propagation in an anisotropic dual-phase lag thermoelastic diffusion medium", Multidiscipline Modeling in Materials and Structures, 10(4), pp 562 – 592.