*International Journal of Latest Trends in Engineering and Technology*
*Vol.(18)Issue(4), pp.001-012*
*DOI: http://dx.doi.org/10.21172/1.184.01*
*e-ISSN:2278-621X*

# POISSON CHEN DISTRIBUTION: PROPERTIES AND APPLICATION

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**Abstract- In this work, we have created a new distribution by using the Poisson-G (generating) family of distribution with baseline distribution as Chen distribution called Poisson Chen (PC) distribution. Some distributional properties of the new distribution are presented, such as the nature of the density function, hazard rate function, cumulative density function and quantile function and the measures of skewness, and kurtosis. To estimate the unknown parameters of PC distribution the maximum likelihood method is introduced and presented the asymptotic confidence intervals based on maximum likelihood estimates. R software is used to perform mathematical computations. The applicability of the new model has been evaluated considering two real datasets and performed the goodness-of-fit attained by the new model via different graphical methods and test statistics. It is found that the new model provided more flexible and a better fit in comparison with some other selected life-time models.**

**Keywords: Poisson family, Chen distribution, Hazard function, MLE.**

## I. INTRODUCTION

In few decades, it has been observed that the many probability distributions have been generated but the real data sets related to engineering, geology, life science, finance, medicine, reliability, life testing, and survival analysis do not always provide a better fit to data set of these distributions. So, the formulation of new generated distributions appears to be necessary to deal with the limitations in these areas. The extended, generalized, and modified models are generated by inserting one or more parameters or making some transformation to the parent distribution. Therefore, the new proposed models will provide a better fit as compared to the challenging models.

A new two-parameter continuous life-time distribution with bathtub-shaped or increasing failure rate function was presented by Chen [1]. The distribution function of Chen distribution can be expressed as

$$\begin{aligned} T(y;\alpha,\beta) &= 1-\exp\left(\beta(1-e^{y^{\alpha}})\right);\ \alpha,\beta>0,\ \mathrm{y}>0 \\ t(y;\alpha,\beta) &= \alpha\beta y^{\alpha-1}e^{y^{\alpha}}\exp\left(\beta(1-e^{y^{\alpha}})\right);\ \alpha,\beta>0,\ \mathrm{y}>0 \end{aligned} \tag{1.1}$$

And its probability density function (PDF) is

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$$t(y;\alpha,\beta)=\alpha\beta y^{\alpha-1}e^{y^{\alpha}}\exp\left(\beta(1-e^{y^{\alpha}})\right);\ \alpha,\beta>0,\ \mathrm{y}>0 \tag{1.2}$$

The motivation to extend the Chen distribution is to set up a flexible model that has revealed the various shapes of the hazard and density functions. Srivastava & Kumar [2] has presented the Chen model and illustrated the MCMC methods for Bayesian inference. Bhatti et al. [3] have created the extended Chen distribution is derived from the generalized Burr-Hatke differential equation and nexus between the exponential and gamma variables. Tarvirdizade and Ahmadpour [4] introduced a new lifetime distribution by compounding of the Weibull and Chen distributions and called Weibull–Chen distribution having increasing and bathtub-shaped hazard rate function and it has constructed. Joshi & Kumar [5] has defined a flexible model called Lindley-Chen distribution using Chen distribution as a base distribution.

The generalized exponential (GE) distribution has introduced by Gupta & Kundu [6] this extended family can accommodate data with increasing and decreasing failure rate functions, also Kus [7] has introduced the two-parameter exponential Poisson (EP) distribution by compounding exponential distribution with zero truncated Poisson distribution with decreasing failure rate. The CDF of PE distribution is,

$$G(x;\beta,\lambda)=\frac{1}{\left(1-e^{-\lambda}\right)}\left[1-\exp\left\{-\lambda\left(1-e^{-\beta x}\right)\right\}\right]\quad ;x>0,(\beta,\lambda)>0$$

While Barreto-Souza and Cribari-Neto [8] have introduced generalized EP distribution having the decreasing or increasing or upside-down bathtub shaped failure rate, which is the generalization of the distribution proposed by Kus [7] adding a power parameter to this distribution. Tracking the similar method, Cancho [9] has introduced a new distribution family also based on the exponential distribution with an increasing failure rate function known as Poisson exponential (PE) distribution. The CDF of PE distribution can be expressed as

$$G(x;\lambda,\theta)=\frac{e^{-\lambda}-\exp\left\{-\theta\left(1-e^{-\lambda x}\right)\right\}}{\left(1-e^{-\lambda}\right)}\quad ;x>0,(\lambda,\theta)>0$$

Louzada-Neto et al [10] has introduced a two-parameter Poisson-exponential with increasing failure rate by using the same approach as used by Cancho [9] under the Bayesian approach. Alkarni and Oraby [11] have presented a new lifetime class with a decreasing failure rate which is obtained by compounding truncated Poisson distribution and a lifetime distribution. The CDF of the Poisson family is given by,

$$D(y;\lambda,\underline{\tau})=1-\frac{1-\exp\left\{-\lambda G(y,\underline{\tau})\right\}}{\left(1-e^{-\lambda}\right)}\quad ;\ \lambda>0$$

Where $\underline{\tau}$ the parameter is space and $G(y,\underline{\tau})$ is the CDF of baseline distribution.

Mahmoudi and Sepahdar [12] have presented a new four-parameter distribution with decreasing, increasing, bathtub-shaped, and unimodal hazard rate called as the exponentiated Weibull–Poisson (EWP) distribution and it has obtained by compounding exponentiated Weibull (EW) and Poisson distributions. The new compounding distribution named the Weibull–Poisson distribution is introduced by Lu & Shi [13] having the failure rate function of shape of increasing, decreasing, upside-down bathtub-shaped or uni-modal. Further Kaviayarasu and Fawaz [14] have made an extensive study on Weibull–Poisson distribution through a reliability sampling plan. Kyurkchiev et al [15] has used the exponentiated exponential-Poisson as the software reliability model. Louzada et al [16] has used different estimation methods to estimate the parameter of exponential-Poisson distribution using rainfall and aircraft data. Joshi and Kumar [17] have presented the Lindley Chen distribution using Chen as base distribution

The different parts of this article are organized as, we present the Poisson Chen distribution with its statistical and mathematical properties in Section 2. We present the maximum likelihood estimation method in Section 3. In Section 4 using a real dataset, we present the estimated values of the model parameters and their corresponding asymptotic confidence intervals and Hassian matrix. Also, we have introduced the different test criteria to assess the applicability of the proposed model. Some concluding remarks are presented in Section 5.

## II. THE POISSON CHEN (PC) DISTRIBUTION

Alkarni and Oraby [11] have introduced the Poisson family and its CDF alternatively may be defined as

$$F\left(t;\lambda,\underline{\upsilon}\right)=1-\frac{1}{\left(1-e^{-\lambda}\right)}\left[1-\exp\left\{-\lambda\left(1-G(t;\underline{\upsilon})\right)\right\}\right] \quad ;t>0,\,\lambda>0 \tag{2.1}$$

And its corresponding PDF is

$$f\left(t;\lambda,\underline{\upsilon}\right)=\frac{1}{\left(1-e^{-\lambda}\right)}\lambda g(t;\underline{\upsilon})\exp\left\{-\lambda\left(1-G(t;\underline{\upsilon})\right)\right\} \quad ;t>0,\,\lambda>0 \tag{2.2}$$

Here $G(t;\underline{\upsilon})$ and $g(t;\underline{\upsilon})$ are the parent cumulative distribution and probability density functions respectively and $\underline{\upsilon}$ be the parameter space of baseline distribution. Now we have taken the Chen distribution as baseline distribution then using cumulative distribution and probability density functions of Chen distribution (1.1) and (1.2) in (2.1) and (2.2) we get the cumulative distribution and probability density functions of Poisson Chen distribution respectively written as

$$F\left(x\right)=1-\frac{1}{1-e^{-\lambda}}\left[1-\exp\left\{-\lambda\; e^{\beta(1-e^{x^{\alpha}})}\right\}\right] \quad ;x>0,(\alpha,\beta,\lambda)>0 \tag{2.3}$$

$$f\left(x\right)=\frac{\alpha\beta\lambda}{1-\exp\left(-\lambda\right)}x^{\alpha-1}e^{x^{\alpha}}\,e^{\beta(1-e^{x^{\alpha}})}\,\exp\left\{-\lambda\; e^{\beta(1-e^{x^{\alpha}})}\right\};\, x>0,(\alpha,\beta,\lambda)>0 \tag{2.4}$$

The Reliability/Survival function of PC distribution is

$$R\left(x\right)=\frac{1}{1-\exp\left(-\lambda\right)}\left[1-\exp\left\{-\lambda\; e^{\beta(1-e^{x^{\alpha}})}\right\}\right] \quad ;x>0,(\alpha,\beta,\lambda)>0 \tag{2.5}$$

HAZARD FUNCTION

Suppose t be endurance time of an item and we desire the probability that it will not survive for an additional time *dt* then, hazard rate function can be expressed as,

$$h\left(t\right)=\frac{\alpha\beta\lambda t^{\alpha-1}e^{t^{\alpha}+\beta(1-e^{t^{\alpha}})}}{\exp\left\{\lambda\; e^{\beta(1-e^{t^{\alpha}})}\right\}-1} \quad ;t>0,(\alpha,\beta,\lambda)>0 \tag{2.6}$$

We have plotted the curve of the PDF and HRF of PC distribution in Figure 1. It has been observed that the shapes of the PC density are arc, positive-skewed, negative-skewed, and symmetrical. The hazard rate function (HRF) for the PC distribution can have various shapes such as increasing, decreasing, decreasing–increasing, increasing–decreasing, reverse J-shaped and bathtub for different values of parameters.

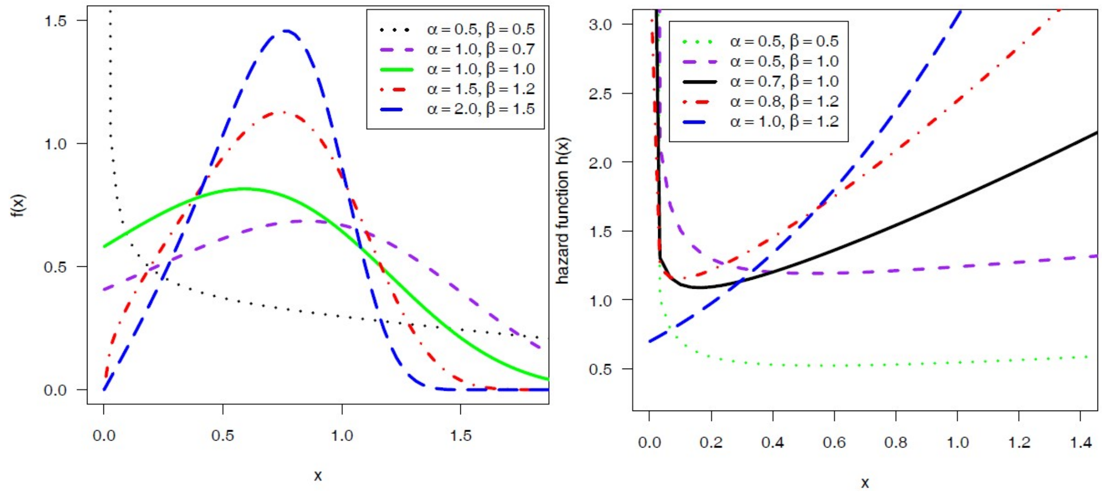


Fig 1. Plots of PDF (left panel) and HF (right panel) for fixed λ and different values of α and β.

QUANTILE FUNCTION OF PC DISTRIBUTION

The quantile function is

$$Q(p)=\left[\ln\left[1-\frac{1}{\beta}\ln\left\{-\frac{1}{\lambda}\ln\left[1-(1-p)\left(1-e^{-\lambda}\right)\right]\right\}\right]\right]^{1/\alpha} \quad ;0<p<1 \tag{2.7}$$

The random numbers of the PC distribution can be generated using the CDF (2.3). Let B denote a uniform random variable in (0, 1), then the simulated values of X can be obtained by setting,

$$x=\left[\ln\left[1-\frac{1}{\beta}\ln\left\{-\frac{1}{\lambda}\ln\left[1-(1-b)\left(1-e^{-\lambda}\right)\right]\right\}\right]\right]^{1/\alpha} \quad ;0<b<1 \tag{2.8}$$

SKEWNESS AND KURTOSIS

The coefficient of skewness of the PC distribution can be obtained as

$$S_k(B)=\frac{Q(0.75)+Q(0.25)-2Q(0.5)}{Q(0.75)-Q(0.25)}, \text{ and}$$

The coefficient of kurtosis given by Moors [18] of the PC distribution can be obtained as

$$K_u(M)=\frac{Q(0.875)-Q(0.625)+Q(0.375)-Q(0.125)}{Q(3/4)-Q(1/4)}$$

## III. ESTIMATION OF MODEL PARAMETERS

Let X be a random variable follows a three-parameter $PC(\alpha,\beta,\lambda)$ having PDF (2.4). The maximum likelihood function of the PC distribution is given by,

$$L(\alpha,\beta,\lambda\,|\,\underline{x})=\frac{\alpha\beta\lambda}{1-e^{-\lambda}}\prod_{k=1}^{n} x_k^{\alpha-1} e^{x_k^{\alpha}}\, e^{\beta(1-e^{x_k^{\alpha}})}\,\exp\left\{-\lambda\, e^{\beta(1-e^{x_k^{\alpha}})}\right\},\ x>0,$$

The log-likelihood function can be written as,

$$l = n\ln(\alpha\beta\lambda) - \ln\left(1-e^{-\lambda}\right) + (\alpha-1)\sum_{k=1}^{n}\ln(x_k) + \sum_{k=1}^{n} x_k^{\alpha} + \beta\sum_{k=1}^{n}(1-e^{x_k^{\alpha}}) - \lambda\sum_{k=1}^{n} e^{\beta(1-e^{x_k^{\alpha}})} \tag{3.1}$$

Differentiating (3.1) with respect to α, β and λ we get,

$$\frac{\partial l}{\partial\alpha} = \frac{n}{\alpha} + \sum_{k=1}^{n}\ln x_k + \sum_{k=1}^{n} x_k^{\alpha}\ln x_k + \beta\sum_{k=1}^{n} x_k^{\alpha} e^{x_k^{\alpha}}\ln x_k + \alpha\beta\sum_{k=1}^{n} x_k^{\alpha} e^{\beta(1-e^{x_k^{\alpha}})+x_k^{\alpha}}\ln x_k$$

$$\frac{\partial l}{\partial\beta} = \frac{n}{\beta} - \sum_{k=1}^{n} e^{x_k^{\alpha}} - \lambda\sum_{k=1}^{n}(1-e^{x_k^{\alpha}})e^{\beta(1-e^{x_k^{\alpha}})}$$

$$\frac{\partial l}{\partial\lambda} = \frac{n}{\lambda} - \frac{ne^{-\lambda}}{1-e^{-\lambda}} - \lambda\sum_{k=1}^{n} e^{\beta(1-e^{x_k^{\alpha}})}$$

By solving these three non-linear equations equating to zero then we get the estimated values of the parameters of the Poisson Chen distribution. With the aid of appropriate computer programming we can solve them numerically.

Suppose the parameter vector $\underset{\sim}{\omega} = (\alpha, \beta, \lambda)$ and the corresponding MLE of $\underset{\sim}{\omega}$ as $\hat{\underset{\sim}{\omega}} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda})$, then the asymptotic normality gives $\left(\hat{\underset{\sim}{\omega}} - \underset{\sim}{\omega}\right) \to N_3\left[0, \left(I\left(\underset{\sim}{\omega}\right)\right)^{-1}\right]$ where $B = I\left(\underset{\sim}{\omega}\right)$ called Fisher's information matrix given by,

$$B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

$$\text{where } B_{11} = \frac{\partial^2 l}{\partial\alpha^2},\ B_{12} = \frac{\partial^2 l}{\partial\alpha\partial\beta},\ B_{13} = \frac{\partial^2 l}{\partial\alpha\partial\lambda}$$

$$B_{21} = \frac{\partial^2 l}{\partial\beta\partial\alpha},\ B_{22} = \frac{\partial^2 l}{\partial\beta^2},\ B_{23} = \frac{\partial^2 l}{\partial\beta\partial\lambda}$$

$$B_{31} = \frac{\partial^2 l}{\partial\lambda\partial\alpha},\ B_{32} = \frac{\partial^2 l}{\partial\beta\partial\lambda},\ B_{33} = \frac{\partial^2 l}{\partial\lambda^2}$$

Further differentiating we get,

$$\frac{\partial^2 l}{\partial\alpha^2} = -\frac{n}{\alpha^2} + \sum_{k=1}^{n} x_k^{\alpha}(\ln x_k)^2 + \sum_{k=1}^{n} x_k^{\alpha}(1+x_k^{\alpha})e^{x_k^{\alpha}}\ln x_k - \beta\lambda\sum_{k=1}^{n} x_k^{\alpha}(\beta x_k^{\alpha}e^{x_k^{\alpha}} - x_k^{\alpha} - 1)(\ln x_k)^2 e^{\beta(1-e^{x_k^{\alpha}})+x_k^{\alpha}}$$

$$\frac{\partial^2 l}{\partial\beta^2} = -\frac{1}{\beta^2} - \lambda\sum_{k=1}^{n}(1-e^{x_k^{\alpha}})^2 e^{\beta(1-e^{x_k^{\alpha}})}$$

$$\frac{\partial^2 l}{\partial \lambda^2} = -\frac{n}{\lambda^2} - \frac{ne^{-\lambda}}{\left(1-e^{-\lambda}\right)^2}$$

$$\frac{\partial^2 l}{\partial \alpha \partial \beta} = \sum_{k=1}^{n} x_k^{\alpha} e^{x_k^{\alpha}} \ln x_k + \lambda \sum_{k=1}^{n} x_k^{\alpha} \ln x_k \left\{1+\beta(1-e^{x_k^{\alpha}})\right\} e^{\beta(1-e^{x_k^{\alpha}})+x_k^{\alpha}}$$

$$\frac{\partial^2 l}{\partial \alpha \partial \lambda} = \beta \sum_{k=1}^{n} x_k^{\alpha} e^{\beta(1-e^{x_k^{\alpha}})+x_k^{\alpha}} \ln x_k$$

$$\frac{\partial^2 l}{\partial \beta \partial \lambda} = \sum_{k=1}^{n} (1-e^{x_k^{\alpha}}) e^{\beta(1-e^{x_k^{\alpha}})}$$

Usually, it is useless that the MLE has asymptotic variance $B^{-1}$ because we don't know $\underline{\omega}$. Hence we approximate the asymptotic variance by putting in the estimated value of the parameters. The common procedure is to use the observed Fisher information matrix $M\left(\hat{\underline{\omega}}\right)$ as an estimate of the information matrix B. Using the Newton-Raphson algorithm to maximize the likelihood creates the observed information matrix and hence the variance-covariance matrix is obtained as,

$$[B]^{-1} = \begin{pmatrix} V(\hat{\alpha}) & \text{cov}(\hat{\alpha},\hat{\beta}) & \text{cov}(\hat{\alpha},\hat{\lambda}) \\ \text{cov}(\hat{\beta},\hat{\alpha}) & V(\hat{\beta}) & \text{cov}(\hat{\beta},\hat{\lambda}) \\ \text{cov}(\hat{\lambda},\hat{\alpha}) & \text{cov}(\hat{\lambda},\hat{\beta}) & V(\hat{\lambda}) \end{pmatrix} \tag{3.2}$$

Hence, approximate 100(1-α) % confidence intervals for α, β and λ can be constructed from the asymptotic normality of MLEs as,

$$\hat{\alpha} \pm Z_{\alpha/2} SE(\hat{\alpha}) \ , \ \hat{\beta} \pm Z_{\alpha/2} SE(\hat{\beta}) \ \text{and,} \ \hat{\lambda} \pm Z_{\alpha/2} SE(\hat{\lambda})$$

where $Z_{\alpha/2} =$ Upper percentile of standard normal variate

## IV. ILLUSTRATION WITH REAL DATA

In this section, we have illustrated the applicability of Poisson Chen distribution using two real datasets used by previous researchers. To compare the potentiality of the proposed model, we have considered the following four distributions.

i) Weibull Extension (WE) Model:

The probability density function of Weibull extension (WE) distribution introduced by Tang et al [19] with three parameters $(\alpha, \beta, \lambda)$ is

$$f_{WE}(x;\alpha,\beta,\lambda) = \lambda\beta\left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left(\frac{x}{\alpha}\right)^{\beta} \mathrm{e}^{-\lambda\alpha\left(\exp\left(\frac{x}{\alpha}\right)^{\beta}-1\right)} \ ; \ x>0$$

$\alpha > 0, \beta > 0$ and $\lambda > 0$

ii) Poisson exponential model (PE)

The probability density function of Poisson–exponential distribution was defined by Louzada-Neto et al [20] also it was used by Rodrigues et al [21] is

$$f(x) = \frac{\beta\lambda}{\left(1-e^{-\lambda}\right)} e^{-\beta x} \exp\left(-\lambda e^{-\beta x}\right); \ \beta>0, \lambda>0, x>0$$

iii) Exponential power (EP) distribution:
The probability density function Exponential power (EP) distribution Smith & Bain [22] is

$$f_{EP}(x) = \alpha\,\lambda^{\alpha}\; x^{\alpha-1}\; e^{(\lambda\, x)^{\alpha}}\; \exp\left\{1 - e^{(\lambda\, x)^{\alpha}}\right\} \quad ;(\alpha, \lambda) > 0, \quad x \geq 0\,.$$

where α and λ are the shape and scale parameters, respectively.

iv) Chen distribution:
Chen [1] has introduced Chain distribution having probability density function (PDF) as

$$f\left(x;\lambda,\theta\right) = \lambda\beta x^{\theta-1} e^{x\theta}\, exp\left\{\lambda\left(1 - e^{x^{\theta}}\right)\right\} \quad ;(\lambda,\theta) > 0, x > 0\,.$$

**Dataset-I**
Badar and Priest [23] have used the data with sample size 63 that represent the strength measured in GPA for single carbon fibers of 10mm in gauge lengths and which are as follows:

1.901, 2.132, 2.203, 2.228, 2.257, 2.350, 2.361, 2.396, 2.397, 2.445, 2.454, 2.474, 2.518, 2.522, 2.525, 2.532, 2.575, 2.614, 2.616, 2.618, 2.624, 2.659, 2.675, 2.738, 2.740, 2.856, 2.917, 2.928, 2.937, 2.937, 2.977, 2.996, 3.030, 3.125, 3.139, 3.145, 3.220, 3.223, 3.235, 3.243, 3.264, 3.272, 3.294, 3.332, 3.346, 3.377, 3.408, 3.435, 3.493, 3.501, 3.537, 3.554, 3.562, 3.628, 3.852, 3.871, 3.886, 3.971, 4.024, 4.027, 4.225, 4.395, 5.020

By using the log-likelihood function (3.1) we have illustrated the maximum likelihood estimates, directly by using R software R Core Team [24] and Dalgaard [25]. We have calculated $\hat{\alpha}$ = 0.5368, $\hat{\beta}$ =1.0024 and $\hat{\lambda}$ = 108.2295 corresponding Log-Likelihood value is -56.2956 from the above dataset. In Table 1 we have demonstrated the MLE's and standard errors (SE) with 95% CI for α, β and λ.

Table 1
MLE, SE and 95% confidence interval

| Parameter | MLE | SE | 95% ACI |
|---|---|---|---|
| **alpha** | 0.5368 | 0.0407 | (0.4571, 0.6165) |
| **beta** | 1.0024 | 0.1292 | (0.7492, 1.2555) |
| **lambda** | 108.2295 | 26.1931 | (56.8911, 159.5679) |

An estimate of the variance-covariance matrix by using MLEs, using equation (3.2) is

$$[B]^{-1} = \begin{pmatrix} 0.00165 & -0.00477 & -0.53689 \\ -0.00477 & 0.01668 & 2.59055 \\ -0.53689 & 2.59055 & 686.0767 \end{pmatrix}$$

The graphs of profile log-likelihood function for the parameters α, β and λ have been depicted in Figure 2 and found that the maximum likelihood estimates can be uniquely determined.

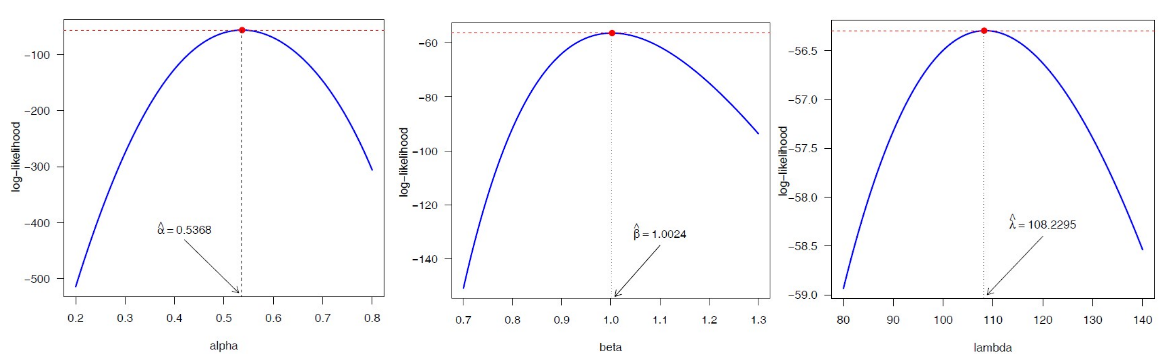

Fig 2. Graph of Profile log-likelihood function for the parameters α, β and λ.

To get the extra information about the goodness-of-fit of the Poisson Chen distribution we have plotted the quantile-quantile (Q-Q) and KS plots in Figure 3. We noticed that the PC model fits the data very well.

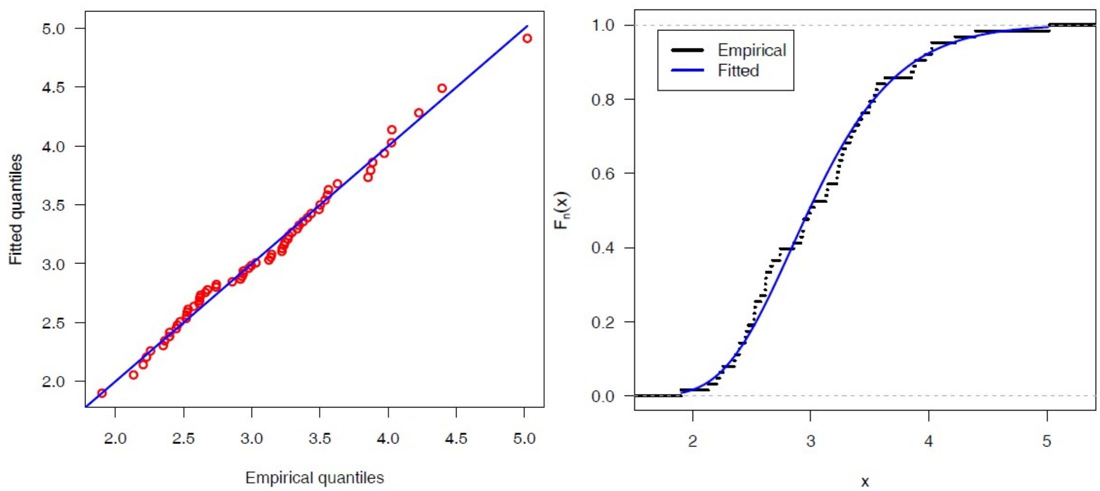


**Figure 3.** The Q-Q plot (left panel) and the KS plot (right panel) of PC distribution

We have calculated the Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike information criterion (CAIC), and Hannan-Quinn information criterion (HQIC) for the assessment of the applicability of the proposed model, which are displayed in Table 2.

Table 2

Log-likelihood (LL), AIC, BIC, CAIC and HQIC

| Model | -LL | AIC | BIC | CAIC | HQIC |
|---|---|---|---|---|---|
| **PC** | 56.2956 | 118.5913 | 125.0207 | 118.9981 | 121.1200 |
| **WE** | 61.9865 | 129.9731 | 136.4025 | 130.3798 | 132.5018 |
| **PE** | 57.2052 | 118.4105 | 122.6967 | 118.6105 | 120.0963 |
| **EP** | 69.3299 | 142.6598 | 146.9461 | 142.8533 | 144.3456 |
| **Chen** | 70.0133 | 144.0265 | 148.3128 | 144.2265 | 145.7124 |

In Figure 4, we have displayed the density function of fitted distributions and the Histogram and Empirical distribution function with the estimated distribution function of PC and some selected models are presented.

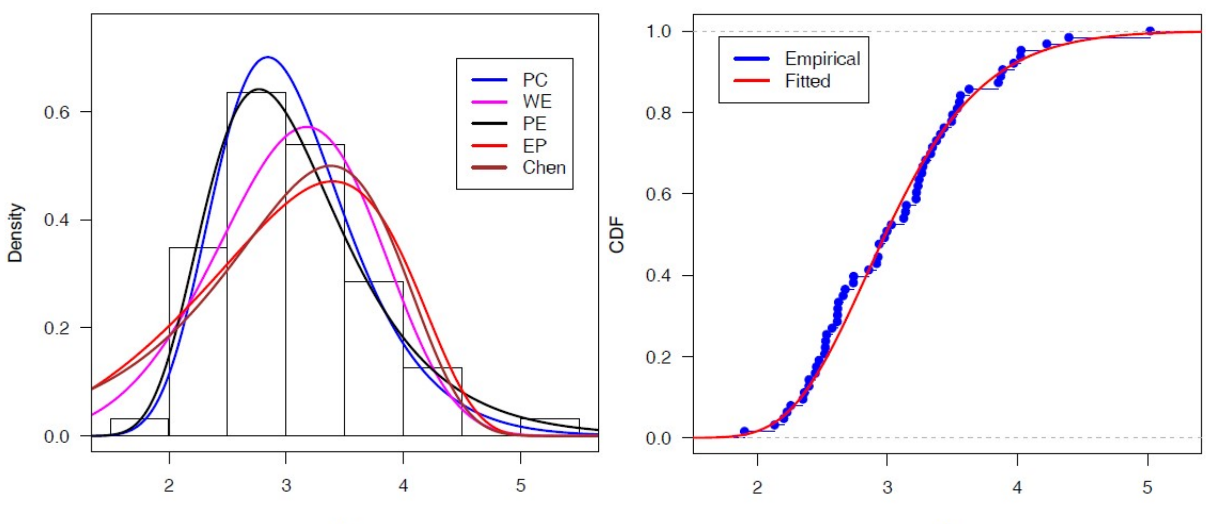


Fig 4. The Histogram and the density function of fitted distributions (left panel) and Empirical distribution function with estimated distribution function (right panel).

We have calculated the value of Kolmogorov-Simnorov (KS), the Anderson-Darling (AD) and the Cramer-Von Mises (CVM) statistics and presented in Table 3 to compare the goodness-of-fit of the PC distribution with other competing distributions. It is observed that the PC distribution has the minimum value of the test statistic and higher *p*-value hence we decide that the PC distribution gets quite better fit and more consistent and reliable results from others taken for evaluation.

**Table 3**

The goodness-of-fit statistics and their corresponding p-value

| Model | *KS(p-value)* | *AD(p-value)* | *CVM(p-value)* |
|---|---|---|---|
| **PC** | 0.0799(0.8151) | 0.0619(0.8039) | 0.3294(0.9142) |
| **WE** | 0.0879(0.7148) | 0.1250(0.4771) | 0.9381(0.3911) |
| **PE** | 0.0635(0.9613) | 0.0543(0.8516) | 0.4044(0.8438) |
| **EP** | 0.1443(0.1452) | 0.3504(0.0978) | 2.3516(0.0595) |
| **Chen** | 0.1534(0.1033) | 0.3123(0.1247) | 2.1928(0.0723) |

**Data set II**

The data gives breaking stress of carbon fibers (in Gba) for 100 observations which was used by Nichols & Padgett [26].

3.70, 2.74, 2.73, 2.50, 3.60, 3.11, 3.27, 2.87, 1.47, 3.11, 4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.90, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96, 2.53, 2.67, 2.93, 3.22, 3.39, 2.81, 4.20, 3.33, 2.55, 3.31, 3.31, 2.85, 2.56, 3.56, 3.15, 2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19, 1.57, 0.81, 5.56, 1.73, 1.59, 2.00, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69, 1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.70, 2.03, 1.80, 1.57, 1.08, 2.03, 1.61, 2.12, 1.89, 2.88, 2.82, 2.05, 3.65

The graphs of profile log-likelihood function for the parameters α, β and λ have been depicted in Figure 5 and found that the maximum likelihood estimates can be uniquely determined.

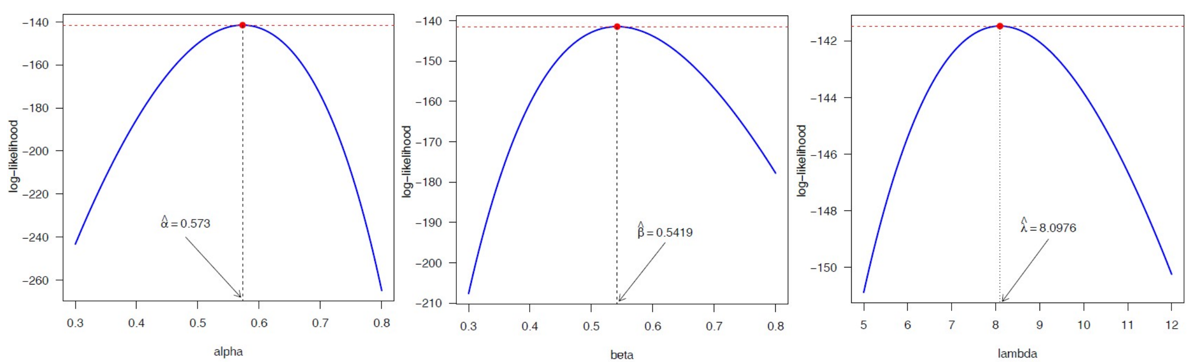


Fig 5. Graph of Profile log-likelihood function for the parameters α, β and λ.

From the above data set, we have obtained $\hat{\alpha}$ = 0.5730, $\hat{\beta}$ = 0.5419 and $\hat{\lambda}$ = 8.0976 corresponding Log-Likelihood value is -141.4765. In Table 4 we have demonstrated the MLE's with their standard errors (SE) and 95% confidence interval for α, β and λ.

**Table 4**

MLE, SE and 95% confidence interval

| Parameter | MLE | SE | 95% ACI |
|---|---|---|---|
| **alpha** | 0.5730 | 0.0510 | (0.4554, 0.6906) |
| **beta** | 0.5419 | 0.1356 | (0.2762, 0.8076) |
| **lambda** | 8.0976 | 2.4463 | (3.3028, 12.8924) |

An estimate of the variance-covariance matrix by using MLEs, using equation (3.2) is

$$[B]^{-1} = \begin{pmatrix} 0.00360 & -0.00773 & -0.12359 \\ -0.00773 & 0.01839 & 0.3094 \\ -0.12359 & 0.3094 & 5.9843 \end{pmatrix}$$

To get the additional information about the goodness of fit of the Poisson Chen distribution we have plotted the Q-Q and KS plots in Figure 6. From Figure 6 it is proven that the PC model fits the data very well.

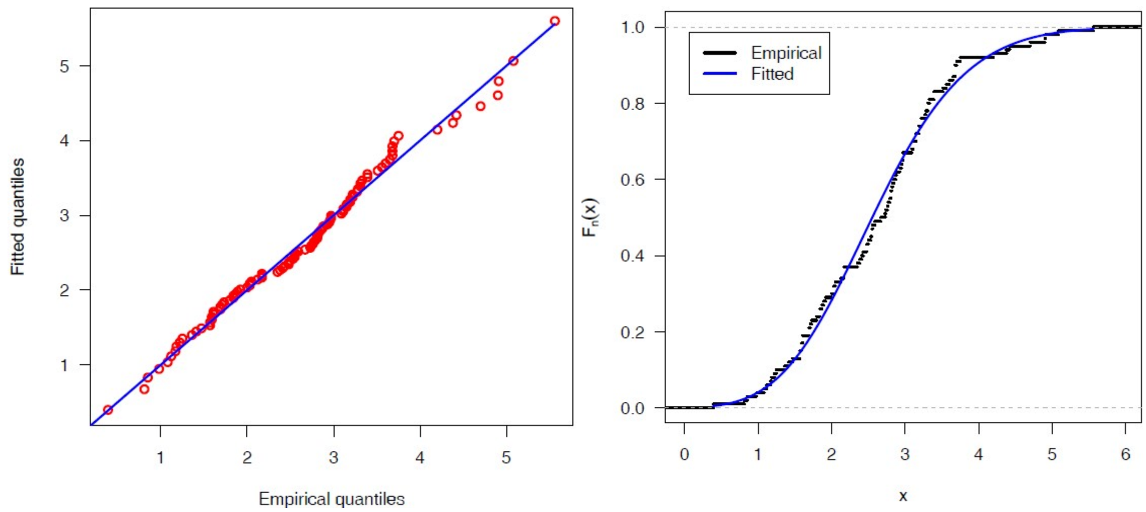


Fig 6. The Q-Q plot (left panel) and the KS plot (right panel) of PC distribution

We have calculated the Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike information criterion (CAIC), and Hannan-Quinn information criterion (HQIC) for the assessment of the applicality of the proposed model, which are displayed in Table 5.

**Table 5**

Log-likelihood (LL), AIC, BIC, CAIC and HQIC

| Model | -LL | AIC | BIC | CAIC | HQIC |
|---|---|---|---|---|---|
| **PC** | 141.4765 | 288.9531 | 296.7686 | 289.2031 | 292.1162 |
| **WE** | 141.5577 | 289.1153 | 296.9309 | 289.3653 | 292.2784 |
| **PE** | 144.2051 | 292.4102 | 297.6205 | 292.5339 | 294.5189 |
| **EP** | 145.9589 | 295.9179 | 301.1282 | 296.0391 | 298.0266 |
| **Chen** | 148.9044 | 301.8089 | 307.0192 | 301.9326 | 303.9176 |

In Figure 7, we have displayed the density function of fitted distributions and the Histogram and Empirical distribution function with the estimated distribution function of PC and some selected models are presented.

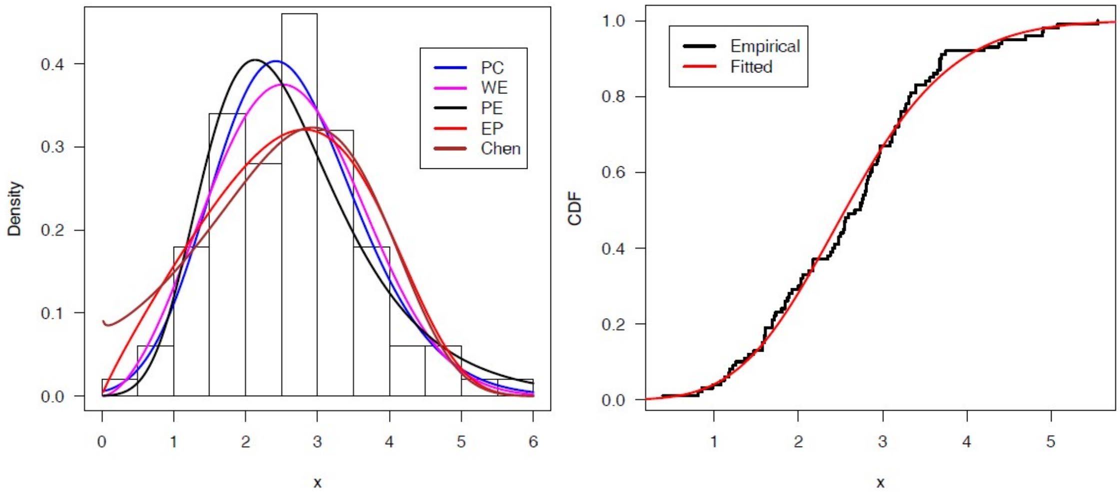


Fig 7. The Histogram and the density function of fitted distributions (left panel) and Empirical distribution function with estimated distribution function (right panel).

We have calculated the value of Kolmogorov-Simnorov (KS), the Anderson-Darling (AD) and the Cramer-Von Mises (CVM) statistics and presented in Table 6 to compare the goodness-of-fit of the PC distribution with other competing distributions. It is observed that the PC distribution has the minimum value of the test statistic and higher *p*-value hence we decide that the PC distribution gets quite better fit and more consistent and reliable results from others taken for evaluation.

Table 6

The goodness-of-fit statistics and their corresponding p-value

| Model | *KS(p-value)* | *AD(p-value)* | *CVM(p-value)* |
|---|---|---|---|
| **PC** | 0.0691(0.7265) | 0.0753(0.7204) | 0.4212(0.8271) |
| **WE** | 0.0607(0.8542) | 0.0635(0.7932) | 0.4212(0.8268) |
| **PE** | 0.0954(0.3229) | 0.1724(0.3284) | 0.9157(0.4044) |
| **EP** | 0.0993(0.2771) | 0.1861(0.2963) | 1.3081(0.2297) |
| **Chen** | 0.0945(0.3336) | 0.2180(0.2353) | 1.6938(0.1364) |

## V. CONCLUDING REMARKS

We have put forward a three-parameter univariate probability distribution called Poisson Chen distribution. A detailed study of some statistical and mathematical properties of the proposed distribution including the derivation of explicit expressions for its reliability function, survival function, hazard function, the quantile function which is useful for calculating partition values and skewness and kurtosis, skewness and kurtosis, and simulation of random numbers from the proposed distribution. The unknown model parameters are estimated using the method of MLE and constructed their corresponding confidence intervals. The graph of the PDF of the proposed distribution has shown that its shape is the skewed model and flexible for modeling real-life data. Also, the graph of the hazard function is monotonically decreasing or increasing according to the value of the model parameters. The performance of the proposed distribution has been evaluated by considering two real-life datasets and the results revealed that the proposed distribution is much flexible as compared to some other fitted distributions.

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