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A STUDY ON PROPERTIES AND REAL DATA APPLICATIONS OF THE LOGISTIC EXPONENTIAL EXTENSION DISTRIBUTION

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Abstract- **- In this article, we have considered a model of three-parameter univariate continuous distribution which is called Logistic exponential extension distribution. We have discussed some statistical properties of the purposed distribution such as the probability density function, cumulative distribution function and hazard rate function, survival function, quantile function, the skewness, and kurtosis measures. Using Maximum likelihood estimation, we have assessed the model parameters of the proposed distribution. By using the maximum likelihood estimate we have constructed the asymptotic confidence interval for the model parameters. We have taken two real data sets for the illustration purposes. The goodness of fit of the proposed distribution is also evaluated by fitting it in comparison with some other existing distributions using two real data sets.**

Keywords – Logistic distribution, Exponential extension distribution, Hazard function, MLE, Survival function

I. INTRODUCTION

Lifetime distributions are generally used to study the length of the life of components of a system, a device, and in general, reliability and survival analysis. Lifetime distributions are often applied in areas such as life science, medical sciences, engineering, biology, insurance, etc. To analyze lifetime data, many continuous probability distributions like Cauchy, gamma, exponential, Weibull have been often used in statistical literature. For a few years, most of the researchers are attracted towards one parameter Logistic distribution for its potential in modeling lifetime data, and it has been seen that this distribution has performed admirably in many areas.

In probability theory and statistics, exponential probability distribution is the continuous memoryless random distribution which has played a significant role in analyses of life testing data. A continuous probability distribution which describes the time elapsed between events in a Poisson process which occurs at a constant average rate occurring independently and continuously is called exponential probability distribution which is a type of the gamma distribution. It is the continuous analog of the geometric distribution.

Inclusion of different shapes such as decreasing, increasing, bathtub-shaped and inverted Bathtub-shaped failure rate in a single model forming a compounded survival model would be beneficial in survival analysis. Such a compounded model would provide desirable properties like considerable flexibility and goodness of fit for fitting a broad variety of lifetime data sets. Furthermore by constructing confidence interval over its model parameter, such survival model may also be used to determine the distribution class from which the data is selected. For these desirable properties, we have introduced the purposed distribution.

The logistic distribution is a univariate continuous distribution which is similar in shape to normal distribution. It is a special case of Tukey lambda distribution. Both its Probability density function and Cumulative distribution

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functions have been used in many different areas such as logistic regression, modeling growth, logit models and neural networks. Application of this distribution is seen in areas such as demography, physical sciences, finance, sports modeling, actuarial sciences etc. In comparison to normal distribution, the logistic distribution has higher kurtosis (heavier tails) so it provides better insight into the likelihood of extreme events and is more consistent with the underlying data.

Let X be a non-negative random variable follows the logistic distribution with shape parameter $\theta > 0$, and its cumulative distribution function is given by

$$G(x;\theta) = \frac{1}{1+e^{-\theta x}}; \quad \theta > 0, x \in \Re \tag{1.3}$$

and its corresponding PDF is

$$g(x;\theta) = \frac{\theta e^{-\theta x}}{\left(1+e^{-\theta x}\right)^2}; \quad \theta > 0, x \in \Re \tag{1.4}$$

Tahir et al. (2016) have introduced the *logistic-X family which is* a new generating family of continuous distributions generated from a logistic random variable [15]. Its probability density function may be reversed-J shaped, symmetrical, right-skewed and, left-skewed and may have bathtub, upside-down bathtub, decreasing and increasing hazard rates shaped. Mandouh (2018) has introduced Logistic-modified Weibull distribution which is flexible for survival analysis as compared to modified Weibull distribution[10]. Joshi & Kumar (2020) have introduced the Lindley exponential power distribution having a more flexible hazard rate function[5]. Lemonte et al. (2015) have introduced 3 parameter extension of the exponential distribution which is quite flexibale and can be used effectively in modeling survival data, reliablility problems, fatigue life studies and hydrological data[9]. It can have increasing, decreasing, constant, bathtub shaped, upside-down bathtub(unimodal) and decreasing-increasing-decreasing hazard rate function. Mansoor et al. (2019) have introduced exponential extension distribution with three parameters where the submodels are the exponential, logistic-exponential and Marshall-Olkin exponential distributions[11]. The distribution has considerable flexibility and its associated probability density function can be unimodal or decreasing. Furthermore, it can have increasing, decreasing, bathtub and upside-down bathtub hazard rates shaped. Chaudhary & Kumar (2020) have introduced the half logistic exponential extension distribution using the parent distribution as exponential extension distribution [3].

Lan & Leemis (2008) has introduced a compounded model called logistic-exponential survival distribution, which consists of Decreasing Failure Rate, Increasing Failure Rate, Upside-Down Bathtub Shaped Failure Rate and Bathtub Shaped Failure rate[8]. This model would be very useful in lifetime modeling. Unlike most distributions in the Upside-Down Bathtub Failure Rate and Bathtub Shaped Failure Rate classes, the logistic–exponential distribution show closed-form density, survival functions, hazard and cumulative hazard. The survival function of the logistic–exponential distribution is

$$S(x;\lambda) = \frac{1}{1+\left(e^{\lambda x}-1\right)^{\alpha}}; \quad \alpha>0, \lambda > 0, x \geq 0 \tag{1.5}$$

We have purposed the new distribution called Logistic exponential extension (LEE) distribution using the same approach used by (Lan & Leemis, 2008). With the aim of achieving more flexibility, we have introduced this distribution by insertion of one more parameter to exponential extension distribution. This helps to achieve a better fit to the lifetime data set. We have presented this distribution with their properties and applicability. Proposed study of the distribution is presented in different section explained as follows. In Section 2 we introduce the Logistic exponential extension (LEE) distribution and their various statistical and mathematical properties. In section 3, the maximum likelihood estimation (MLE) is used to estimate the model parameters. We have constructed asymptotic confidence intervals using the observed information matrix for the Maximum Likelihood estimation (MLE).In Section 4 the application and capability of Logistic exponential extension (LEE) distribution has been studied by analyzing two real dataset. Here, the goodness of fit of the proposed distribution is compared through a real data set to some well-known existing distributions. Finally, in Section 5 we present conclusion to the study.

II. THE LOGISTIC EXPONENTIAL EXTENSION(LEE) DISTRIBUTION

We have purposed the new distribution called Logistic exponential extension (LEE) distribution using the same approach used by (Lan & Leemis, 2008)[8]. In this study we have taken the exponential extension (EE) (Joshi, 2015)[6] as baseline distribution with CDF and PDF respectively as follows,

$$G(x)=1-\exp\left\{-\lambda x e^{-\beta/x}\right\} \quad ; x>0, \lambda>0, \beta>0 \tag{2.1}$$

$$g(x)=\lambda\left(1+\frac{\beta}{x}\right)e^{-\beta/x} \exp\left\{-\lambda x e^{-\beta/x}\right\} \quad ; x>0, \lambda>0, \beta>0 \tag{2.2}$$

Let X be a non-negative random variable with a positive scale parameter λ and a positive shape parameters α and β then Cumulative Distribution Function(CDF) of logistic exponential extension distribution can be defined as

$$F(x)=1-\frac{1}{1+\left[\exp\left\{-\lambda x e^{-\beta/x}\right\}-1\right]^{\alpha}}; \quad (\alpha,\beta,\lambda)>0,\ x>0 \tag{2.3}$$

The Probability Density Function(PDF) of logistic exponential extension distribution is defined as

$$f(x)=\frac{\alpha\lambda(1+\beta/x)e^{-\beta/x}\exp\left\{-\lambda x e^{-\beta/x}\right\}\left[\exp\left\{-\lambda x e^{-\beta/x}\right\}-1\right]^{\alpha-1}}{\left\{1+\left[\exp\left\{-\lambda x e^{-\beta/x}\right\}-1\right]^{\alpha}\right\}^{2}}; \quad (\alpha,\beta,\lambda)>0,\ x>0 \tag{2.4}$$

This CDF function resembles the log logistic CDF function with the second term of the denominator being changed in its base to an exponential extension function, so we named it as logistic exponential extension distribution. We denote it as LEE distribution.

A. Reliability function/Survival function

The reliability function of LEE distribution is defined as,

$$R(x)=1-F(x)=\frac{1}{1+\left[\exp\left\{-\lambda x e^{-\beta/x}\right\}-1\right]^{\alpha}}; \quad (\alpha,\beta,\lambda)>0,\ x>0 \tag{2.5}$$

B. Hazard function

The hazard rate function of LEE distribution is defined as,

$$h(x)=\frac{f(x)}{R(x)}=\frac{\alpha\lambda(1+\beta/x)e^{-\beta/x}\exp\left\{-\lambda x e^{-\beta/x}\right\}\left[\exp\left\{-\lambda x e^{-\beta/x}\right\}-1\right]^{\alpha-1}}{\left\{1+\left[\exp\left\{-\lambda x e^{-\beta/x}\right\}-1\right]^{\alpha}\right\}} \tag{2.6}$$

In Figure 1, we have shown the plots of the PDF and hazard rate function of LEE distribution for different values of α, β and λ.

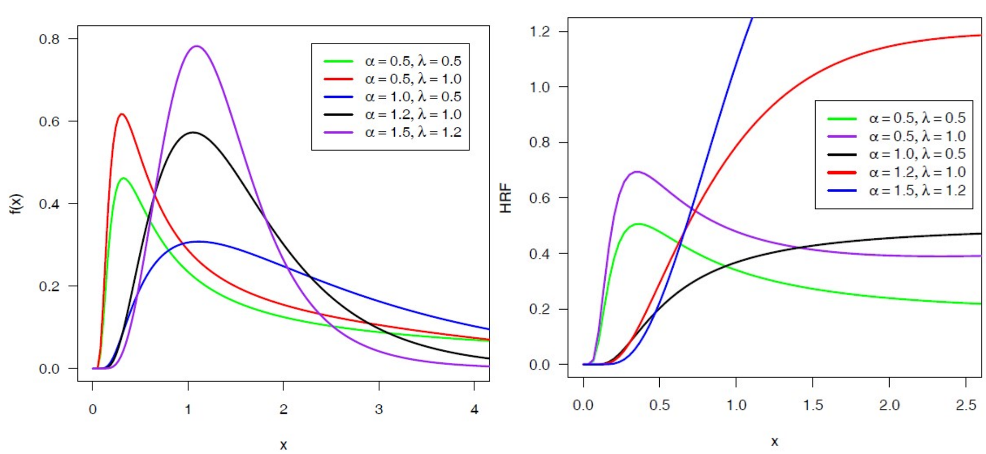

Figure 1. Plots of PDF (left panel) and hazard function (right panel) for different values of α, β and λ.

C. Quantile function

The Quantile function of Logistic exponential power distribution can be expressed as

$$\ln(x)-\frac{\beta}{x}-\ln\left[-\frac{1}{\lambda}\ln\left\{\left(\frac{p}{1-p}\right)^{1/\alpha}+1\right\}\right]=0;\quad 0<p<1 \tag{2.7}$$

D. *Skewness and Kurtosis:*

The Bowley's coefficient of Skewness is the measure of Skewness based on quartiles which can be expressed as,

$$Skewness=\frac{Q(0.75)+Q(0.25)-2Q(0.5)}{Q(0.75)-Q(0.25)} \text{ and} \tag{2.8}$$

As defined by (Moors,1988), the coefficient of Kurtosis based on octiles is,

$$K_u(M)=\frac{Q(0.875)-Q(0.625)+Q(0.375)-Q(0.125)}{Q(3/4)-Q(1/4)}, \tag{2.9}$$

III. METHODS OF ESTIMATION

In this section, the parameters of the proposed distribution are estimated by applying the well-known method the maximum likelihood method.

Maximum Likelihood Estimates

The most commonly used method for the estimation of the model parameter is the maximum likelihood method. (Casella & Berger, 1990)[2]. Let, $x_1,x_2,...,x_n$ is a random sample from $LEE(\alpha,\beta,\lambda)$ and the likelihood function, $L(\alpha,\beta,\lambda)$ is given by,

$$L(\psi;x_1,x_2...x_n)=f(x_1,x_2,...x_n/\psi)=\prod_{i=1}^{n}f(x_i/\psi)$$

$$L(\alpha,\beta,\lambda)=\alpha\lambda\prod_{i=1}^{n}\frac{(1+\beta/x_i)e^{-\beta/x_i}\exp\left\{-\lambda x_i e^{-\beta/x_i}\right\}\left[\exp\left\{-\lambda x e^{-\beta/x_i}\right\}-1\right]^{\alpha-1}}{\left\{1+\left[\exp\left\{-\lambda x e^{-\beta/x_i}\right\}-1\right]^{\alpha}\right\}^{2}};\ (\alpha,\beta,\lambda)>0,\ x>0$$

Now log-likelihood density is

$$\ell(\alpha,\beta,\lambda\,|\,\underline{x})=n\ln(\alpha\lambda)+\sum_{i=1}^{n}\ln(1+\beta/x_i)-\beta\sum_{i=1}^{n}1/x_i-\lambda\sum_{i=1}^{n}x_i e^{-\beta/x_i}+(\alpha-1)\sum_{i=1}^{n}\ln\left\{\exp\left(-\lambda x_i e^{-\beta/x_i}\right)-1\right\}$$
$$-2\sum_{i=1}^{n}\ln\left\{1+\left[\exp\left(-\lambda x_i e^{-\beta/x_i}\right)-1\right]^{\alpha}\right\} \tag{3.1}$$

Differentiating (3.1) with respect to α, β and λ we get,

$$\frac{\partial l}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\ln\left(e^{-\lambda x_i e^{-\beta/x_i}}-1\right)-2\sum_{i=1}^{n}\frac{\left(e^{-\lambda x_i e^{-\beta/x_i}}-1\right)^{\alpha}\ln\left(e^{-\lambda x_i e^{-\beta/x_i}}-1\right)}{1+\left(e^{-\lambda x_i e^{-\beta/x_i}}-1\right)^{a}}$$

$$\frac{\partial l}{\partial \lambda}=\frac{n}{\lambda}-\sum_{i=1}^{n} x_i e^{-\beta/x_i}+(\alpha-1)\sum_{i=1}^{n}\frac{x_i e^{-\lambda x_i e^{-\beta/x_i}-\beta/x_i}}{e^{-\lambda x_i e^{-\beta/x_i}}-1}+2\alpha\sum_{i=1}^{n}\frac{x_i e^{-\lambda x_i e^{-\beta/x_i}-\beta/x_i}\left(e^{-\lambda x_i e^{-\beta/x_i}}-1\right)^{\alpha-1}}{1+\left(e^{-\lambda x_i e^{-\beta/x_i}}-1\right)^{a}}$$

The values of unknown model parameters α, β and λ can be obtained by equating above three linear equations to zero and solving them simultaneously for α, β and λ. We obtain the corresponding the maximum likelihood estimate $\hat{\alpha},\ \hat{\beta}\ and\ \hat{\lambda}$ of the parameters α, β and λ.To obtain the estimated value of α, β and λ, we have used statistical computing software such as R, Mathematica, Matlab etc for maximization of equation(3.1)

We have to calculate the observed information matrix to construct the confidence interval estimation of α, β and λ and for testing of the hypothesis,. The observed information matrix for α, β and λ can be obtained as,

$$C=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Where

$$C_{11}=\frac{\partial^2 l}{\partial \alpha^2},\ C_{12}=\frac{\partial^2 l}{\partial \alpha \partial \beta},\ C_{13}=\frac{\partial^2 l}{\partial \alpha \lambda}$$

$$C_{21}=\frac{\partial^2 l}{\partial \beta \partial \alpha},\ C_{22}=\frac{\partial^2 l}{\partial \beta^2},\ C_{23}=\frac{\partial^2 l}{\partial \beta \partial \lambda}$$

$$C_{31}=\frac{\partial^2 l}{\partial \lambda \partial \alpha},\ C_{32}=\frac{\partial^2 l}{\partial \beta \partial \lambda},\ C_{33}=\frac{\partial^2 l}{\partial \lambda^2}$$

Let $\Omega=(\alpha,\beta,\lambda)$ denote the parameter space and the corresponding MLE of Ω as $\hat{\Omega}=(\hat{\alpha},\hat{\beta},\hat{\lambda})$, then $\left(\hat{\Omega}-\Omega\right)\rightarrow N_3\left[0,\left(C(\Omega)\right)^{-1}\right]$ where $C(\Omega)$ is the Fisher's information matrix. Using the Newton-Raphson algorithm to maximize the likelihood creates the observed information matrix and hence the variance-covariance matrix is obtained as,

$$\left[C(\Omega)\right]^{-1}=\begin{pmatrix} \mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha},\hat{\beta}) & \mathrm{cov}(\hat{\alpha},\hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha},\hat{\beta}) & \mathrm{var}(\hat{\beta}) & \mathrm{cov}(\hat{\beta},\hat{\lambda}) \\ \mathrm{cov}(\hat{\alpha},\hat{\lambda}) & \mathrm{cov}(\hat{\beta},\hat{\lambda}) & \mathrm{var}(\hat{\lambda}) \end{pmatrix} \tag{3.2}$$

Hence from the asymptotic normality of MLEs, approximate 100(1-α) % confidence intervals for α, β and λ can be constructed as,

$$\hat{\alpha}\pm Z_{\alpha/2}SE(\hat{\alpha})\ ,\ \hat{\beta}\pm Z_{\alpha/2}SE(\hat{\beta})\ \text{and,}\ \hat{\lambda}\pm Z_{\alpha/2}SE(\hat{\lambda})$$

Where $Z_{\alpha/2}$ is the upper percentile of standard normal variate

IV. REAL DATA APPLICATIONS

In this section we have taken two real data sets for the illustration purposes which are as follows,

Data Set 1

The data given below represents the fatigue life of 6061-T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per seconds (cps) which consists of 101 observations with maximum stress per cycle 31,000 psi. This data set was originally analyzed by Birnbaum and Saunders (1969) [1].

70, 90, 96, 97, 99, 100, 103, 104, 104, 105, 107, 108, 108, 108, 109, 109, 112, 112, 113, 114, 114, 114, 116, 119, 120, 120, 120, 121, 121, 123, 124, 124, 124, 124, 124, 128, 128, 129, 129, 130, 130, 130, 131, 131, 131, 131, 131, 132, 132, 132, 133, 134, 134, 134, 134, 134, 136, 136, 137, 138, 138, 138, 139, 139, 141, 141, 142, 142, 142, 142, 142, 142, 144, 144, 145, 146, 148, 148, 149, 151, 151, 152, 155, 156, 157, 157, 157, 157, 158, 159, 162, 163, 163, 164, 166, 166, 168, 170, 174, 196, 212

The MLEs are calculated directly by using optim() function in R software (R Core Team, 2020)[14] and (Ming, 2019)[12] by maximizing the likelihood function (3.1). By maximizing the likelihood function in (3.1) we have obtained $\hat{\alpha}$ = 1.7919, $\hat{\beta}$ = 418.0473, $\hat{\lambda}$ = 0.1211 and corresponding Log-Likelihood value is l = -455.3564. In Table 1 we have demonstrated the MLE's with their standard errors (SE) and 95% confidence interval for α, β, and λ.

Table 1
MLE and SE and 95% confidence interval for α, β and λ

Parameter	MLE	SE	95% ACI
alpha	1.7919	0.1561	(1.4860, 2.0979)
beta	418.0473	4.767	(408.704, 427.3906)
lambda	0.1211	0.0092	(0.1031, 0.1391)

.
We have displayed the graph of the profile log-likelihood function of α, β, and λ in Figure 2 and observed that the MLEs are unique.

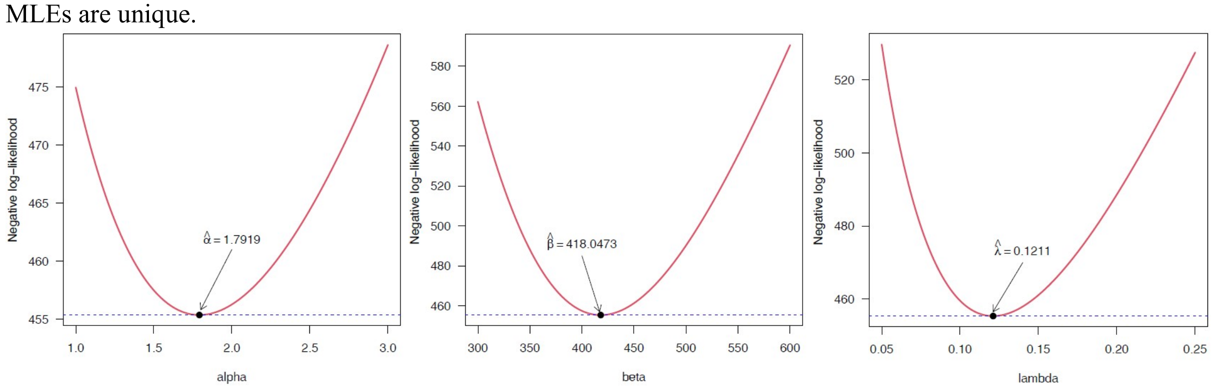

Figure 2. Graph of profile log-likelihood function of α, β, and λ.

In Figure 3 we have presented the P-P plot (empirical distribution function against theoretical distribution function) and Q-Q plot (empirical quantile against theoretical quantile).

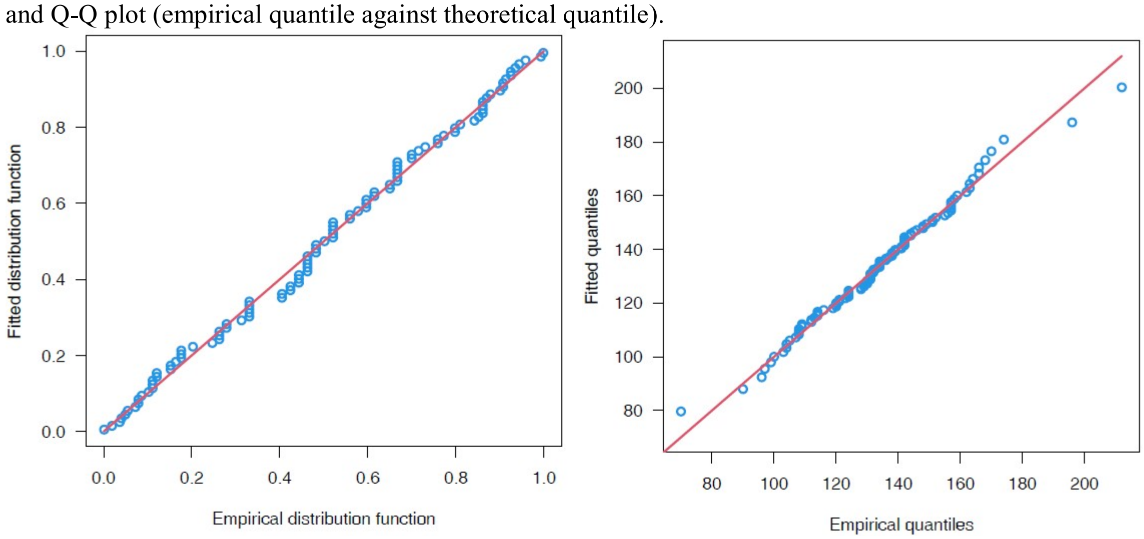

Figure 3. The P-P plot (left panel) and Q-Q plot (right panel) of LEE distribution

For the goodness of fit purpose we use negative log-likelihood (-LL), Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike Information criterion (AICC) and Hannan-Quinn information criterion (HQIC), statistic to select the best model among selected models. The expressions to calculate AIC, BIC, AICC and HQIC are listed below:

a) $AIC = -2l(\hat{\theta}) + 2k$

b) $BIC = -2l(\hat{\theta}) + k\log(n)$

c) $AICC = AIC + \dfrac{2k(k+1)}{n-k-1}$

d) $HQIC = -2l(\hat{\theta}) + 2k\log\left[\log(n)\right]$

where k is the number of parameters and n is the size of the sample in the model under consideration. Further, in order to evaluate the fits of the LEE distribution with some selected distributions we have taken the Kolmogorov-Simnorov (KS), the Anderson-Darling (W) and the Cramer-Von Mises (A^2) statistic. These statistics are widely used to compare non-nested models and to illustrate how closely a specific CDF fits the empirical distribution of a given data set. These statistics are calculated as

$$KS = \max_{1\le i\le n}\left(d_i - \frac{i-1}{n}, \frac{i}{n} - d_i\right)$$

$$W = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\ln d_i + \ln\left(1-d_{n+1-i}\right)\right]$$

$$A^2 = \frac{1}{12n} + \sum_{i=1}^{n}\left[\frac{(2i-1)}{2n} - d_i\right]^2$$

where $d_i = CDF(x_i)$; the x_i's being the ordered observations.

To illustrate the goodness of fit of the Logistic exponential extension distribution, we have taken some well-known distribution for comparison purpose which are listed blew,

A. Generalized Rayleigh(GR) distribution

The probability density function of Generalized Rayleigh (GR) distribution (Kundu & Raqab, 2005)[7] is

$$f_{GR}\,(\mathrm{x};\,\alpha,\lambda) = 2\,\alpha\,\lambda^2\,\mathrm{x}\,\mathrm{e}^{-(\lambda x)^2}\left\{1-\mathrm{e}^{-(\lambda x)^2}\right\}^{\alpha-1} \qquad ;(\alpha,\lambda)>0, x>0$$

Here α and λ are the shape and scale parameters respectively.

B. Chen distribution

The probability density function of Chen distribution (Chen, 2000) is given by[4],

$$f_{CN}\left(x;\lambda,\beta\right) = \lambda\,\beta\,\;x^{\beta-1}\,e^{x^{\beta}}\,exp\left\{\lambda\left(1-e^{x^{\beta}}\right)\right\} \quad ;(\lambda,\beta)>0, x>0.$$

C. *GENERALIZED EXPONENTIAL (GE) DISTRIBUTION*

The probability density function of generalized exponential distribution (Gupta & Kundu, 1999)

$$f_{GE}\left(x;\alpha,\lambda\right) = \alpha\,\lambda\,e^{-\lambda\,x}\left\{1-e^{-\lambda\,x}\right\}^{\alpha-1} \quad ;(\alpha,\lambda)>0, x>0.$$

D. *Exponential power (EP) distribution*

The probability density function Exponential power (EP) distribution (Smith & Bain, 1975) is

$$f_{EP}(x) = \alpha\,\lambda^{\alpha}\,x^{\alpha-1}\,e^{(\lambda\,x)^{\alpha}}\,\exp\left\{1-e^{(\lambda\,x)^{\alpha}}\right\} \quad ;(\alpha,\lambda)>0, \quad x\ge 0.$$

where α and λ are the shape and scale parameters, respectively.
For the judgment of potentiality of the proposed model we have presented the value of Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike information criterion (AICC) and Hannan-Quinn information criterion (HQIC) which are presented in Table 2.

Table 2
Log-likelihood (LL), AIC, BIC, AICC and HQIC

Model	-LL	AIC	BIC	AICC	HQIC
LEE	455.3564	916.7128	924.5581	916.9602	919.8888
GR	457.3766	918.7532	923.9835	918.8757	920.8706
GE	463.7324	931.4648	936.6951	931.5873	933.5822
Chen	467.0598	938.1196	943.3499	938.2421	940.2370
EP	476.7897	957.5794	962.8096	957.6994	959.6967

The Histogram and the density function of fitted distributions and Empirical distribution function with estimated distribution function of LEE and some selected distributions are presented in Figure 4.

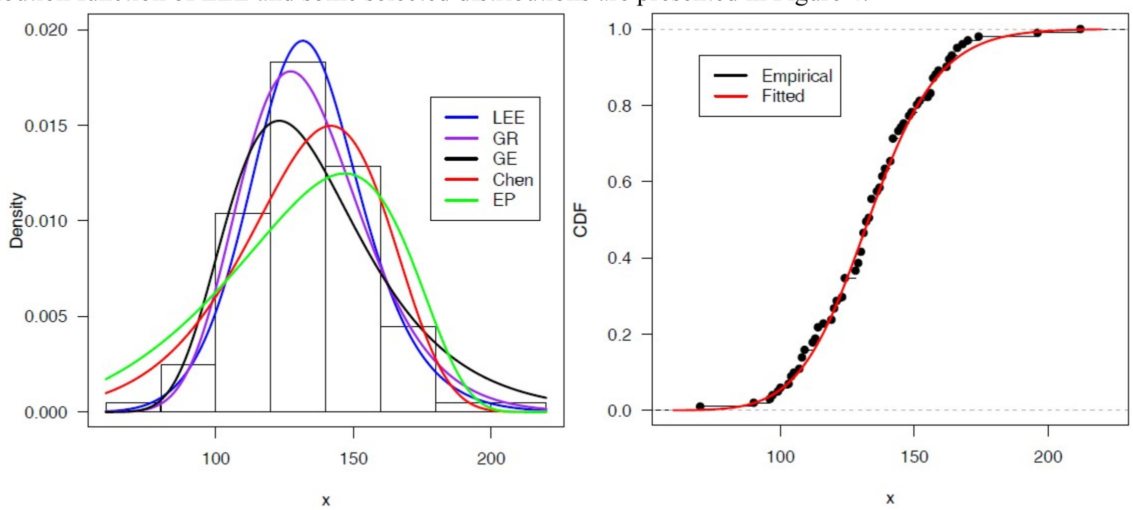

Figure 4. The Histogram and the density function of fitted distributions (left panel) and Empirical distribution function with estimated distribution function (right panel).

In comparision of the goodness-of-fit of the LEE distribution with other competing existing distributions we have presented the p-values of the Anderson-Darling (AD), the Cramer-Von Mises (CVM), and Kolmogorov-Simnorov (KS) statistics in Table 3. It is seen that the LEE distribution has the minimum value of the test statistic and higher *p*-value which shows that the LEE distribution gets the best fit for the used real data set and more consistent and reliable results than others existing distribution taken for comparison.

Table 3
The goodness-of-fit statistics and their corresponding p-value

Model	*AD(p-value)*	*CVM(p-value)*	*KS(p-value)*
LEE	0.0402(0.9331)	0.2631(0.9630)	0.0584(0.8811)
GR	0.1050(0.5620)	0.6033(0.6445)	0.0901(0.3850)
GE	0.3112(0.1257)	2.0724(0.0840)	0.1066(0.2014)
Chen	0.2960(0.1386)	2.0769(0.0835)	0.1102(0.1718)
EP	0.6942(0.0130)	4.5057(0.0050)	0.1378(0.0433)

Data Set 2
The data below are from an accelerated life test of 59 conductors, (Nelson & Doganaksoy, 1995)[13]. The failures can occur in microcircuits because of the movement of atoms in the conductors in the circuit; this is referred to as electro-migration. The failure times are in hours, and there are no censored observations.

6.545, 9.289, 7.543, 6.956, 6.492, 5.459, 8.120, 4.706, 8.687, 2.997, 8.591, 6.129, 11.038, 5.381, 6.958, 4.288, 6.522, 4.137, 7.459, 7.495, 6.573, 6.538, 5.589, 6.087, 5.807, 6.725, 8.532, 9.663, 6.369, 7.024, 8.336, 9.218, 7.945, 6.869, 6.352, 4.700, 6.948, 9.254, 5.009, 7.489, 7.398, 6.033, 10.092, 7.496, 4.531, 7.974, 8.799, 7.683, 7.224, 7.365, 6.923, 5.640, 5.434, 7.937, 6.515, 6.476, 6.071, 10.491, 5.923.

. By maximizing the likelihood function in (3.1). We have obtained $\hat{\alpha} = 1.1363$, $\hat{\beta} = 23.5526$, $\hat{\lambda} = 3.0617$ and corresponding Log-Likelihood value is l = -111.4168. In Table 4 we have demonstrated the MLE's with their standard errors (SE) for α, β, and λ.

Table 4
MLE and SE for α, β and λ

Parameter	MLE	SE
alpha	1.1363	0.5083
beta	23.5526	11.8539
lambda	3.0617	5.1212

.

We have displayed the graph of the profile log-likelihood function of α, β, and λ in Figure 5 and observed that the MLEs are unique.

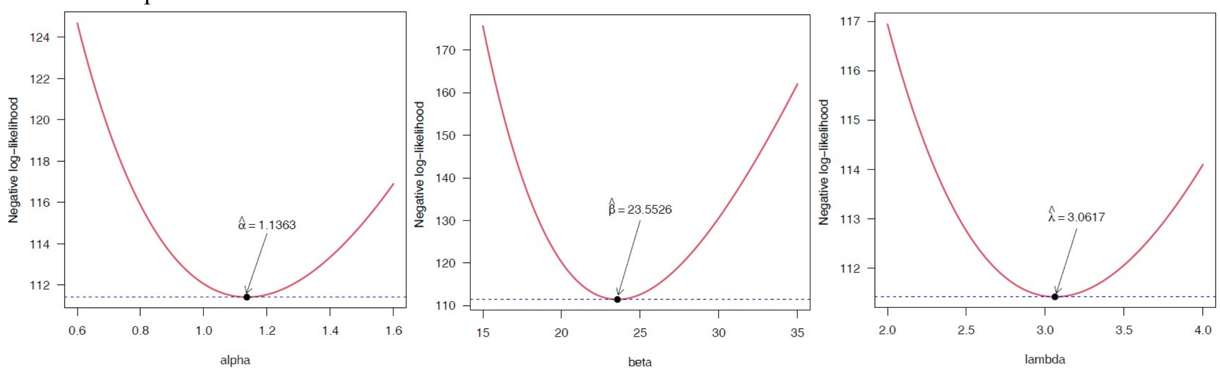

Figure 5. Graph of profile log-likelihood function of α, β, and λ.

In Figure 6 we have presented the P-P plot (empirical distribution function against theoretical distribution function) and Q-Q plot (empirical quantile against theoretical quantile).

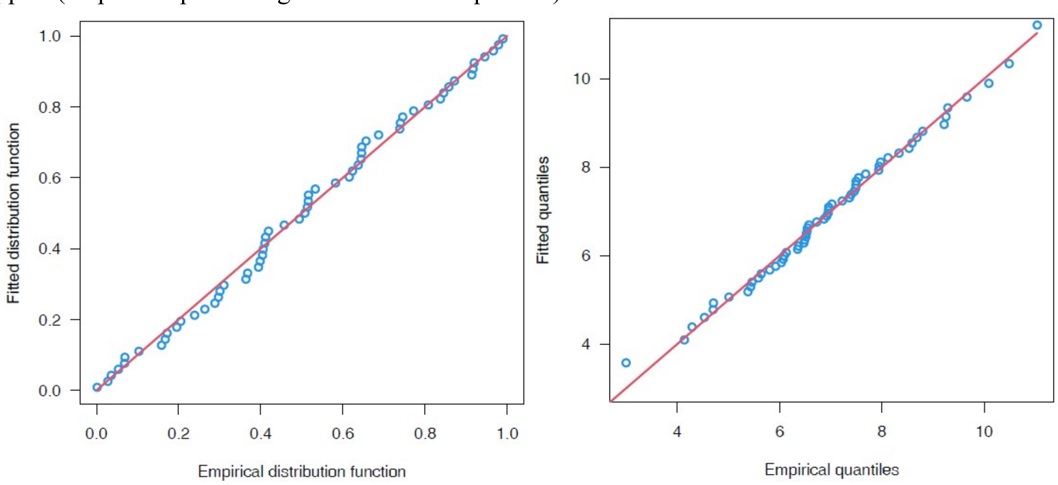

Figure 6. The P-P plot (left panel) and Q-Q plot (right panel) of LEE distribution

For the judgment of potentiality of the proposed model we have presented the value of Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike information criterion (AICC) and Hannan-Quinn information criterion (HQIC) which are presented in Table 5.

Table 5
Log-likelihood (LL), AIC, BIC, AICC and HQIC

Model	-LL	AIC	BIC	AICC	HQIC
LEE	111.4168	228.8337	235.0663	229.2700	231.2666
GR	111.8717	227.7434	231.8984	227.9576	229.3653
GE	114.9473	233.8946	238.0497	234.1089	235.5166
Chen	116.3874	236.7748	240.9299	236.9891	238.3968
EP	116.5015	237.0029	241.1580	237.2098	238.6249

The Histogram and the density function of fitted distributions and Empirical distribution function with estimated distribution function of LEE and some selected distributions are presented in Figure 7.

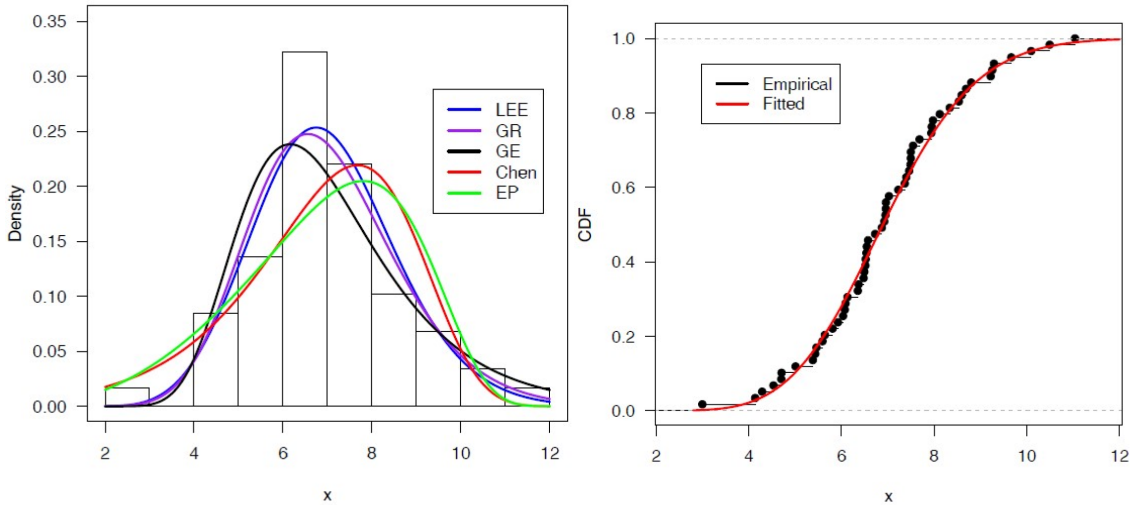

Figure 7. The Histogram and the density function of fitted distributions (left panel) and Empirical distribution function with estimated distribution function (right panel).

In comparision of the goodness-of-fit of the LEE distribution with other competing existing distributions we have presented the p-values of the Anderson-Darling (AD), the Cramer-Von Mises (CVM), and Kolmogorov-Simnorov (KS) statistics in Table 6. It is seen that the LEE distribution has the minimum value of the test statistic and higher *p*-value which shows that the LEE distribution gets the best fit for the used real data set and more consistent and reliable results than others existing distribution taken for comparison.

Table 6
The goodness-of-fit statistics and their corresponding p-value

Model	*AD(p-value)*	*CVM(p-value)*	*KS(p-value)*
LEE	0.0302(0.9764)	0.1858(0.9938)	0.0586(0.9801)
GR	0.0433(0.9171)	0.2554(0.9674)	0.0741(0.8788)
GE	0.1173(0.5079)	0.7368(0.5282)	0.1042(0.5103)
Chen	0.1913(0.2855)	1.1741(0.2774)	0.1238(0.3006)
EP	0.2398(0.2021)	1.3735(0.2098)	0.1365(0.2021)

V. CONCLUSION

In this paper, we have purposed a new three-parameter univariate continuous distribution called Logistic exponential extension (LEE) distribution and some of statistical properties of the LEE distribution including the shapes of the cumulative distribution, probability density function, survival function, hazard rate functions, quantile function, the skewness, and kurtosis measures are studied. We found that the proposed model has considerable flexibility and is inverted bathtub shaped hazard function. The maximum likelihood estimation (MLE) methods is used to estimate the model parameters. Two real data sets are analyzed to explore the applicability, suitability and flexibility of the proposed distribution and found that the proposed model is quite better fit than other well-known competitive lifetime models such as Generalized Rayeleigh, Chen, Generalized Exponential and Exponential Power distribution taken into consideration. We hope this purposed model may be an alternative in the field of survival analysis, probability theory and applied statistics.

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