

A NOTE ON MENSURATION

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Abstract- Under plane geometry area of circle is evaluated. The cone and sphere are mapped from curved to plane geometry and areas are calculated. Volume pyramid is decided and it is used to calculate volumes of cone and sphere. The entire computation is based on elementary mathematics rather than calculus. Such method is not available elsewhere.

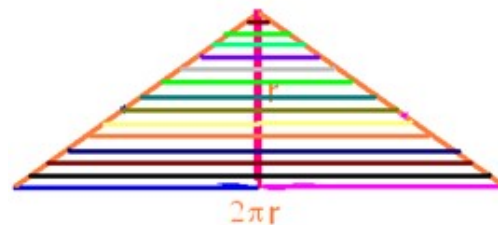
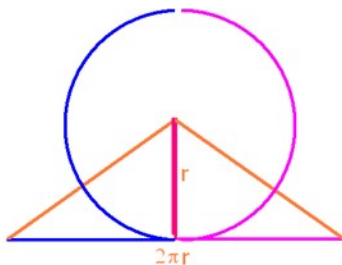
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I. INTRODUCTION

The Great Mathematician Aryabhata invented that perimeter of a circle and its diameter has a constant value, we call it 'π' now. Geometrical shapes and bodies occupy a major space in physical and mathematical problems. At elementary level subject is delivered as to remember the formulas. At intermediate level calculus is adopted to derive them. Here I will present analytical consideration at very elementary level so that there should no more need to memorize or forget such interesting things.

II. AREA OF CIRCLE AND CONE

A circular area[1] can be regarded as composed of infinite number of circular paths ranging from centre to the perimeter. Now take a set A of radii and set B of semi-perimeter and plot on graph, we will obtain a straight line. Hence if we take linear perimeters as base and radius as height, then similar triangles are produced.



Hence, we can calculate the area very easily as follows

$$\text{area of circle} = \text{area of triangle} = \frac{1}{2} \times \text{height} \times \text{base} = \frac{1}{2} \times r \times 2\pi r = \pi r^2 \quad \dots (1)$$

Now we consider a right circular cone and set A belongs to the points on curved surface of the cone and map them into set B on the plane surface.

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Of course, plane surface not only one but it is collection of infinite plane surfaces of triangular shape whose height is lateral side of the cone and sum of bases equals the perimeter of the base.

Curved Surface Area-

$$\Delta A = \frac{1}{2} l \Delta s \Rightarrow A = \Sigma \Delta A = \frac{1}{2} l (2\pi r) = \pi r l \quad \dots(2)$$

III. AREA OF SPHERE

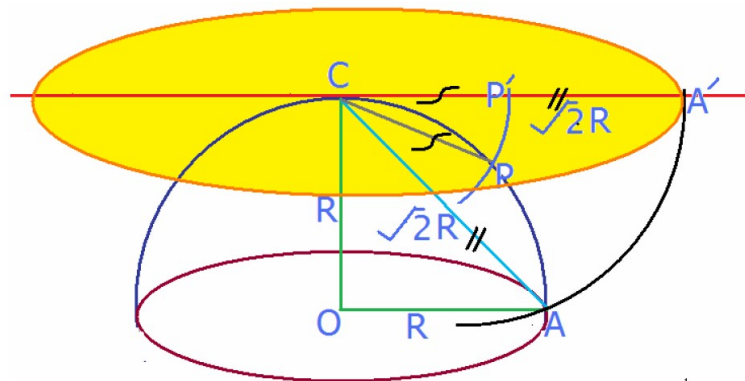
We consider a hemisphere and define set A as points on curved surface. Now we map these points on plane figure of set B in the following manner.

Using Pythagoras theorem, it is easy to find that the distance

$$CA = \sqrt{2} \cdot R$$

- Any chord cuts the circle or sphere at most at two points only.
- The concentric spheres or circles of distinct radii never intersect.

Drawing the arcs of radius $CA=CA'$, $CP=CP'$we can map all the points of curved surface to the corresponding points on the circle of radius CA and centre at C . Moreover, this mapping has one-to-one correspondence uniquely. *



Thus, all the area of curved surface of hemisphere converts to area of circle with radius $CA = \sqrt{2} \cdot R$.

$$\begin{aligned} \therefore \text{area of curved surface of hemisphere} &= \pi (\sqrt{2} \cdot R)^2 = 2\pi R^2 \\ \text{Area of Complete Sphere} &= 4\pi R^2 \end{aligned} \quad \dots(3)$$

IV. VOLUME OF PYRAMID

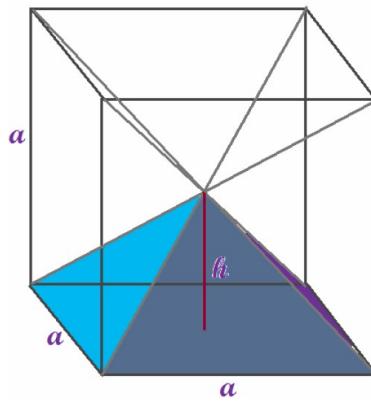
Body diagonals with faces make six pyramids within the cube. As a general ansatz we assume that the volume must possess area of base and height as factors in the expression.

$$\begin{aligned} \therefore \text{volume of pyramid} &= k (\text{area of base}) (\text{height}) \\ \Rightarrow \text{volume of six pyramids} &= 6k a^2 h = \text{volume of cube} \end{aligned}$$

$$\Rightarrow 6k a^2 a/2 = a^3 \Rightarrow k=1/3$$

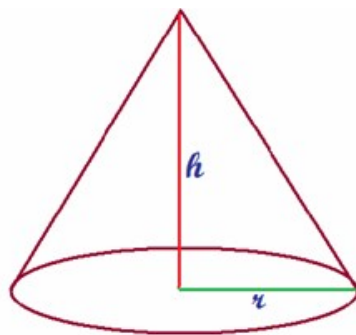
$$\therefore \text{volume of pyramid} = (1/3)(\text{area of base})(\text{height})$$

$$\Rightarrow \text{volume of six pyramids} = 6k a^2 h = \text{volume of cube}$$



V. VOLUME OF CONE

Now using this idea, we find volume of the cone.



$$V = \frac{1}{3} \pi r^2 h$$

VI. VOLUME OF SPHERE

Now construct cones of infinitesimal base and height equal to radius within the sphere. Sum of the base areas will be equal to area of the sphere and height is the same.

Therefore, volume will be $V = (1/3) (\text{area of base}) (\text{height})$

$$V = \frac{1}{3} (4\pi r^2) r = \frac{4}{3} \pi r^3 \quad \dots(5)$$

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* Here the references are just to match the results appeared in the paper , the entire methodology is newly developed by the corresponder.