



UNIFIED DEFINITION OF STATISTICAL MEANS AND APPLICATION TO CIRCUIT ANALYSIS

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Abstract- Arithmetic, Harmonic and Geometric Means are analyzed to conform in a unique definition. The new term eset is introduced. The paper exhibits fundamental development in the area of mathematics, statistics physics.

Keywords – Statistical Means, eset, electric circuits.

I. INTRODUCTION

The problem of unification of basics laws or theories or definitions always remains of immense importance and keen interest. The mathematician, physicists and scientists devote their triumphs to formulate the ideas in least possible conceptual dimensions. The conciseness is the soul of any theoretical framework. In this direction we devote our study to unify Statistical Means and their application to circuit analysis.

II. ANALYSIS OF DEFINITIONS OF STATISTICAL MEANS

The composition \oplus of identical elements leads to new operation \otimes defined as

$$a \oplus a \oplus a \oplus \dots \dots \dots f\text{-times} = a \otimes f \quad \dots(1)$$

The operation \otimes is distributive over the composition \oplus . if M is some representative or mean of the elements, the each element can be replaced by it.

$$\begin{aligned} & (x_1 \oplus x_1 \oplus x_1 \oplus \dots \dots \dots f_1 - \text{times}) \oplus (x_2 \oplus x_2 \oplus x_2 \oplus \dots \dots \dots f_2 - \text{times}) \oplus \dots \\ & = (M \oplus M \oplus M \oplus \dots \dots \dots f_1 - \text{times}) \oplus (M \oplus M \oplus M \oplus \dots \dots \dots f_2 - \text{times}) \oplus \dots \end{aligned}$$

This concept leads to the following formula

$$(x_1 \otimes f_1) \oplus (x_2 \otimes f_2) \oplus \dots \dots = M \otimes (f_1 \oplus f_2 \oplus \dots) \quad \dots(2)$$

We define eset of n-items as

$$\Xi = \{ \{x_1, x_2, x_3, \dots \} \} \quad \dots(3)$$

The eset is different from the set in the sense that here any element may reappear. All elements of eset have the same unit and dimensions. In this way it escapes from group properties in usual sense.

The arithmetic mean of $\Xi = \{ \{x_1, x_2, x_3, \dots \} \}$ can be evaluated if we choose composition as simple addition and then associated operation becomes the multiplication. Using Eq. (2) we obtain

$$\langle x \rangle = \frac{f_1 x_1 + f_2 x_2 + \dots}{f_1 + f_2 + \dots} = A \quad \dots(4)$$

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The harmonic mean of $\Xi = \{x_1, x_2, x_3, \dots\}$ can be evaluated if we choose composition as simple addition and then associated operation becomes the multiplication. Using Eq. (2) we obtain

$$\langle x^{-1} \rangle = \frac{f_1 x_1^{-1} + f_2 x_2^{-1} + \dots}{f_1 + f_2 + \dots} = H^{-1} \quad \dots(5)$$

The geometric mean of $\Xi = \{x_1, x_2, x_3, \dots\}$ can be evaluated if we choose composition as simple multiplication and then associated operation becomes the exponent. Using Eq. (2) we obtain

$$\langle X \rangle^{f_1 + f_2 + \dots} = x_1^{f_1} x_2^{f_2} \dots = G^{f_1 + f_2 + \dots} \quad \dots(6a)$$

If these quantities are dimensionless, then after taking logarithms we see

$$\log G = \frac{f_1 \log x_1 + f_2 \log x_2 + \dots}{f_1 + f_2 + \dots} \quad \dots(6b)$$

To achieve unification we introduce an operator Λ in such a way that:

$$\Lambda_1 x = x \text{ (identity operator)}, \quad \Lambda_2 x = x^{-1} \text{ (reciprocal operator)}, \quad \Lambda_3 x = \log x \text{ (logarithmic operator)}$$

$$\langle \Lambda_1 x \rangle = \frac{f_1 \Lambda_1 x_1 + f_2 \Lambda_1 x_2 + \dots}{f_1 + f_2 + \dots} = A,$$

$$\langle \Lambda_2 x \rangle = \frac{f_1 \Lambda_2 x_1 + f_2 \Lambda_2 x_2 + \dots}{f_1 + f_2 + \dots} = H^{-1},$$

$$\langle \Lambda_3 x \rangle = \frac{f_1 \Lambda_3 x_1 + f_2 \Lambda_3 x_2 + \dots}{f_1 + f_2 + \dots} = \log G$$

This formulation leads to the general definition of Mean as

$$\langle \Lambda x \rangle = \frac{\sum_i f_i \Lambda x_i}{\sum_i f_i} \quad \dots(7)$$

This idea can be further extended simply. Let there be two mathematical operations \oplus & \otimes with identity elements θ and e . Then above definition can be depicted as

$$\langle \Lambda x \rangle = \frac{(f_1 \otimes \Lambda x_1) \oplus (f_2 \otimes \Lambda x_2) \oplus (f_3 \otimes \Lambda x_3) \oplus \dots}{(f_1 \otimes e) \oplus (f_2 \otimes e) \oplus (f_3 \otimes e) \oplus \dots} \quad \dots(8)$$

The more general form will be

$$(\Lambda x_1 \otimes f_1) \oplus (\Lambda x_2 \otimes f_2) \oplus \dots = \langle \Lambda x \rangle \otimes (f_1 \oplus f_2 \oplus \dots) \quad \dots(9)$$

III. APPLICATION TO ELECTRIC CIRCUITS

We apply our ideas to series and parallel combinations of resistors and capacitors. The equivalent resistance or capacitance can be computed in terms of arithmetic or harmonic mean.

The only ansatz we keep in mind is

$$\eta^2 = 1, \quad (Z^\eta)^\eta = Z, \quad Z_{eq} = (Z_1^\eta + Z_2^\eta + Z_3^\eta + \dots + Z_N^\eta)^\eta = N \langle Z^\eta \rangle \quad \dots(10)$$

For resistors in series (parallel) combination $\eta=+1(-1)$ while for capacitors in series (parallel) combination $\eta=-1(+1)$

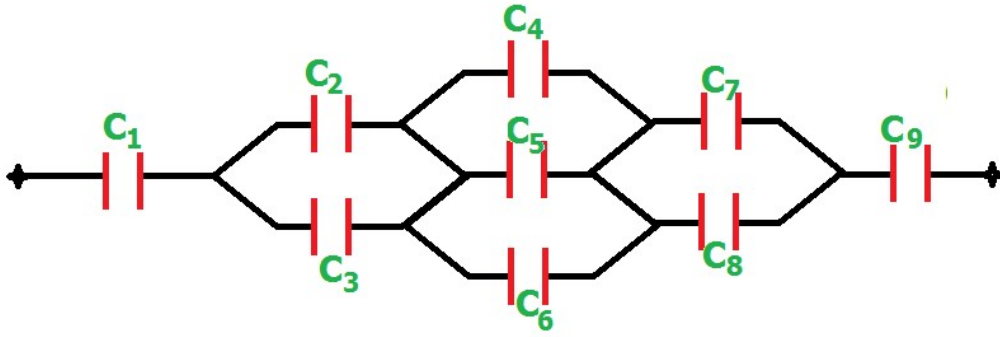


Figure 1

The eset for fig.1 = $S\{C_1, P\{C_2, C_3\}, P\{C_4, C_5, C_6\}, P\{C_7, C_8\}, C_9\}$.

$$C_{eq} = \left(C_1^{-1} + (C_2 + C_3)^{-1} + (C_4 + C_5 + C_6)^{-1} + (C_7 + C_8)^{-1} + C_9^{-1} \right)^{-1} \quad \dots(11)$$

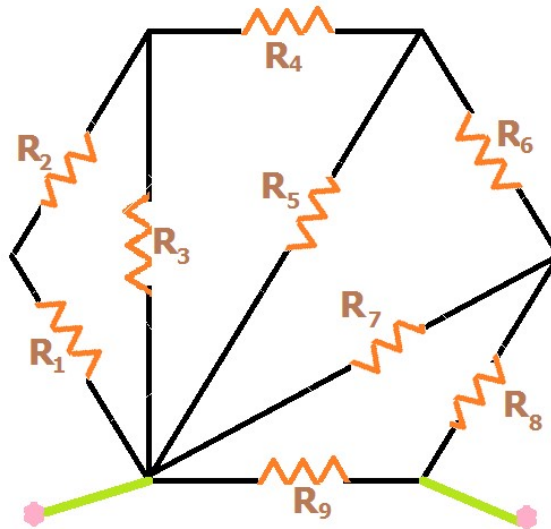


Figure 2

The eset for fig.1 = $P\{S\{P\{S\{P\{S\{P\{S\{R_1, R_2\}, R_3\}, R_4\}, R_5\}, R_6\}, R_7\}, R_8\}, R_9\}\}$.

IV. DISCUSSION AND CONCLUSION

In context to Eq. (5) we see that different sets of $\{\Lambda, \oplus, \otimes\}$ will lead to new type of Means. This paper will open new route to interdisciplinary research. To represent a circuit by a mathematical expression is quite satisfactory and the concept of eset is a new useful enough to represent a circuit in eset form.

REFERENCES

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* Here the references are just to match the results appeared in the paper , the entire methodology is newly developed by the corresponder.