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FUZZY INVENTORY MODEL WITH CONSIGNMENT POLICY

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Abstract- Reducing the cost and achieving significant profit are the key factors for every successful business and industrial sector. A consignment contract approach may be a fruitful combination to achieve a profitable business. This model deals with a single-period inventoryproblem with fuzzy price with a consignment policy. The consignment policy is an agreement between any two parties, named as the consignor and the consignee. Both parties carry some parts of the holding cost instead of one. Stochastical demand and transportation cost are also considered in the problem.

Key words – Manufacturer,Retailer,Profit

I.INTRODUCTION

Newsvendor problem is very famous and important in the field of inventory and supply chain management. The organization has to decide how much tostock of products in inventory before observing the customer's demand. In case of demand uncertainty, stock ownersometimes faces overstock or understock situation. As the products are perishable, overage products begin to deteriorate.Arrow, Harris, and Marschak(1951) developed the formulation of newsvendor problem. Newsvendorproblem plays an important role for short life cycle products like agricultural, dairy products as per authors Pal, Sana, and Chaudhuri (2013)).

Consignment policy is one of the most efficient policies in supply chain management. The upstream party (e.g., manufacturer) is referred to the consignor andthe consignee denotes the downstream party (e.g., retailer). Consignment stock is an inventory, which is retained by thedownstream party but upstream party holds the ownership of this inventory. No fund transfer is to be taken place untiland unless an item is sold from the retailer's stock. The retailer receives per unit commission per each sold item. The manufacturer also agrees to pay afixed fee to the retailer. As the retailer's space is used to stock the inventory, the holding cost is divided between theboth parties.

This paper considers a newsvendor model under distribution-free approach with a comparison between traditional and consignment policy. Section 2 describes literature review .Section 3 consists of problem definition with assumptions and notation to develop the model, the mathematical model. Section 4 includes the solution methodology with algorithm. includes the numerical examples to validate the mathematical model and Sensitivity analysis. Section 5consists of the concluding remarks

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II. LITERATURE REVIEW

According to Chen and Liu (2008), the inventory carrying cost for the retailer is reduced in caseof consignment contract than traditional system. Sarkar et al. (2015b) observed a supply chainmodel with unequal lot sizes, variable set-up cost and carbon emission cost. A partially backordered supply chain wasstudied by Park (2015) and with variable backorderwas investigated by Sarkar (2016) using inspections to detect defective products, and discount policy for fixed lifetimeproducts Kusukawa and Alozawa (2015); Sarkar et al. (2017a). Sarkar, Hur, and Sarkar (2017b) studied a supply chain model with along with set up cost variable production rate and time dependent holding cost. Supply chain model in fuzzy environment (Khalili-Damghani and Ghasemi 2016) is more realistic.In make-to-order policy, for satisfying customer's demand, manufacturer produces goods in case of insufficient inventory and delivers these to the retailer (Mahajan, Radas, and Vakharia 2002). According to Valentini and Zavanella (2003), inventory carrying or holding cost mainly consists of two components as financial component and operational component. In traditional policy, total carrying cost has to be incurred by the retailer. On the other hand, in case of consignment policy, the financial component is incurred by the manufacturer and the operational component is carried by the vendor (Chen and Liu 2008). Adida and Ratisoontorn (2011) investigated how competition among the retailers affects the supply chain decisions and profits under different consignment contracts. Gerchak and Khmelnitsky (2003) examined a consignment policy Yi and Sarker (2013) developed an integrated inventory model underconsignment stock policy with the buyer's space limitation and controllable lead time. Yu et al. (2012) considered a singlemanufacturermulti-retailer consignment model with generalized demand distribution and solved the model using a normaland an exponential distribution. Sarker (2014), Wang, Lee, and Cang (2012) modelled a consignment inventory system with deterioratingitems when buyer has warehouse capacity constraint. Sarkar et al. (2016a, 2016b) investigated the effect of variable transportationcost along with carbon emission cost in a three-echelon supply chain system. A gamestrategic model was developed by Golmohammadi et al. (2016) under VMI supply chain. Jafari and Seifbarghy (2016)optimized bi-objective multi-echelon multi-product supply chain model using new pareto-based approaches. Ouyang, Chen, and Chang(2002) investigated an inventory model with quality improvement, set-up cost reduction under normally distributed and distribution-free lead time demand. Ouyang, Wu, and Ho (2004) ,Sarkar and Majumder (2013) Sarkar et al. (2015, 2015a) developed a distribution-free newsvendor model with nonlinear holding cost and set up cost. Sarkarand Moon (2014) investigated an imperfect production process with variable backorder costs to improve quality andreduction of set-up cost. Shin et al. (2016) developedin a continuous review inventory model having variable lead time demand, which was solved by distribution-freeapproach and optimised the model under transportation discounts strategy. Braglia, Castellano, and Song (2017) discussedon stochastic joint-replenishment problem. Sarkar et (2018) discussed distribution free newsvendor model with consignment policy and retailer's royalty reduction keeping scope for consideration of transportation cost in future.

Fuzzy set theory has been used to model systems that are hard to define precisely. Zadeh (1965) stated that most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive system.

A. Triangular fuzzy number –

Triangular fuzzy number (TFN) is a fuzzy number represented with three points as $\tilde{A} = (a_1, a_2, a_3)$, where a_1 and a_3 denote the lower and upper limits of support of a fuzzy set $\tilde{A}$. The membership value of the x denoted by $\mu_{\tilde{A}}(x)$, $x \in R^+$, can be calculated according to the following formula (Figure. 1).

$$\mu_{\tilde{A}}(x) = \begin{cases} 0; & x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1}; & a_1 < x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}; & a_2 < x < a_3 \\ 0; & x \ge a_3 \end{cases}$$

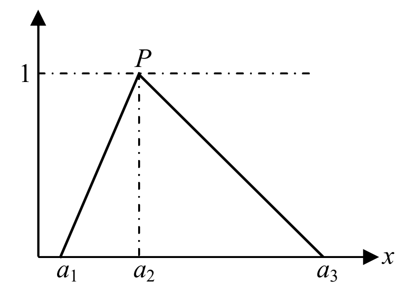

Figure1.

B. Defuzzification –

The system characteristics are described by membership function; it preserves the fuzziness of input information. However, the designer would prefer one crisp value for one of the system characteristics rather than fuzzy set. In order to overcome this problem, we defuzzify the fuzzy values of system characteristic by using the Yager's (1981) formula. If $\tilde{A} = (a1,a2,a3)$, then its crisp value is defined as:

$$\text{crisp}(\tilde{A}) = h(\tilde{A}) = \frac{3a_2 + a_3 - a_1}{3}$$

III.Problem Definition, Assumptions and Notation

This section provides definition of the proposed model, assumptions and notation .

A.Problem definition –

A comparison between a traditional and a consignment policy on a supply chain model with single manufacturer and single retailer., the study considers known mean and a known standard deviation. Figure 1 represents the supply chain.

To establish a consignment contract, the negotiation between manufacturer and retailer must be initiated properly. In consignment policy, manufacturer retains the ownership of products. Nevertheless, under certain condition, retailer has to pay a specific amount of fund (royalty) to the manufacturer which can be a barrier to achieve the contract between two parties.

B. Assumptions –

- A single-period supply chain model is considered under make-to-order policy. Manufacturer must produce and deliver items, without sufficient inventory on hand to satisfy customer's demand (Mahajan, Radas, and Vakharia, 2002).
- Customer demand is considered as random, with a known mean and a known standard deviation
- In traditional system, total inventory carrying cost is incurred by retailer while manufacturer only incurs the manufacturing cost. In consignment policy Manufacturer offers a consignment contract to retailer under make-to-order production system.
- In consignment contract, manufacturer holds the ownership of products even if products are shifted to retailer's warehouse/ outlet. The holding cost is split into two parts, financial and operational, in consignment contract. Financial part is carried by manufacturer and operational part is incurred by retailer.
- The price of a product is not precise in real world hence the price is considered in fuzzy term.in traditional system transportation cost is born by the retailer, where as in consignment system it is born by the manufacturer.

Figure2.Supply chain

C. Notation –

Variables:

Q	order quantity (units)
a	commission for each item sold(rS/unit)
A	fixed cost given by the manufacturer to die buyer (rs)
D	demand of the customer (units)

Parameters:

p	fuzzy retail price of the product(S/unit)
h_r^{TS}	holding cost of retailer under traditional system(rs/unit/unit time)
h_r^{CP}	holding cost of retailer under consignment policy(rs/unit/unit time)
h_m^{CP}	holding cost of manufacturer under consignment policy(rs/unit/unit time)
s_r	goodwill loss per unit item for retailer(rs/unit)
s_m	goodwill loss per unit item for manufacturer (rs/unit)
c	manufacturing cost for unit item(rs/unit)
μ	mean of buyer's demand
σ	standard deviation of buyer'sdemand
w	wholesale price(rs/unit)
t	transportation cost rs/unit

Other notations:

$E(.)$	mathematical expectation
$x+$	maximum value of x and 0
π_r^{TS}	retailer's profit for traditionalpolicy
π_m^{TS}	manufacturer's profit for traditional policy
π_r^{CP}	retailer's profit for consignment policy
π_m^{CP}	manufacturer's profit for consignment policy
π_j^{CP}	joint profit (retailer | manufacturer) for consignment policy

D. Mathematical model –

There are two models based on traditional policy and consignment policy.

E. Traditional Policy (TP) –

In traditional policy, the total inventory carrying/holding cost is incurred by the retailer. The retailer purchases the total lot from the manufacturer at a wholesale price, retains the ownership of it and sells the product to the customer at a selling price. The manufacturer takes no responsibility of goods and incurs no cost for holding after delivering items to the retailer's end. The retailer's total profit for traditional system is

$$\pi_r^{TS} = \begin{cases} pD - wQ - tQ - h_r^{TS}(Q-D)^+; \ D \le Q \\ (p-w-t)Q - s_r E(D-Q)^+; \ Q < D \end{cases}$$

The retailer's expected profit can be written as

$$E\left(\pi_r^{TS}\right) = \begin{cases} p\mu - wQ - tQ - h_r^{TS}(Q-D)^+; \ D \le Q \\ (p-w-t)Q - s_r E(D-Q)^+; \ Q < D \end{cases} \quad (1)$$

F. Distribution approach –

As no assumption for the probability distribution of the random demand $D$is considered, a class Ω of cumulative distribution is assumed. One can suppose any cumulative distribution function F of D with mean μ and standard deviation σ such that $F \in \Omega$.

i) $$E(Q-D)^{+} \le \frac{1}{2}\left[\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)\right] (2)$$

ii) $$E(Q-D)^{+} \le \frac{1}{2}\left[\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right] (3)$$

Using (2) and (3), the expected profit of (1) can be written as

$$E\left(\pi_r^{TS}\right)=p(\mu+Q)-wQ-tQ-h_r^{TS}\left\{\frac{[\sigma^2+(Q-\mu)^2]^{\frac{1}{2}}-(\mu-Q)}{2}\right\}-s_r\left\{\frac{[\sigma^2+(Q-\mu)^2]^{\frac{1}{2}}-(Q-\mu)}{2}\right\}$$

$$=p(\mu+Q)-wQ-tQ-\frac{h_r^{TS}+s_r}{2}[\sigma^2+(Q-\mu)^2]^{\frac{1}{2}}-\frac{h_r^{TS}+s_r}{2}(Q-\mu) (4)$$

The revenue of the manufacturer is wQ and the cost incurred by the manufacturer is the manufacturing cost. The expected profit of the manufacturer is

$$E\left(\pi_m^{TS}\right)=wQ-cQ=(w-c)Q (5)$$

In order to maximise the expected total profit of the retailer, one can obtain the derivative of the profit function in (4) with respect to Q as

$$\frac{\partial E\left(\pi_r^{Ts}\right)}{\partial Q}=p-w-\frac{h_r^{TS}+s_r}{2}[\sigma^2+(Q-\mu)]^{-\frac{1}{2}}(Q-\mu)-\frac{h_r^{TS}-s_r}{2}$$

Equating the above equation to zero, one can obtain

$$Q_r^{*}=\mu+\frac{\sigma\Gamma}{\sqrt{1-\Gamma^2}} \text{ where } \Gamma=\frac{2(p-w)-\left(h_r^{TS}-s_r\right)}{h_r^{TS}+s_r} \quad \text{[see Appendix-B].}(6)$$

Using (6), (5) can be written as

$$E\left(\pi_m^{TS}\right)=(w-c)Q_r^{*}=(w-c)\left[\mu+\frac{\sigma\Gamma}{\sqrt{1-\Gamma^2}}\right] (7)$$

For any real Q and $\mu > 0$, $E\left(\pi_r^{TS}\right)$ is bounded above

From (6), one can see that for real Q, $\Gamma < 1$ must holds which implies $\Gamma^2 << 1$, i.e.,

$$\frac{2(p-w)-\left(k_r^{TS}-s_r\right)}{h_r^{TS}+s_r}<1 \text{ or, } h_r^{TS}>(p-w) (8)$$

Now using (8), the profit function in (4) can be written as

$$E\left(\pi_r^{TS}\right)<p(\mu+Q)-wQ-tQ-(p-w)\left\{\frac{[\sigma^2+(Q-\mu)^2]^{\frac{1}{2}}-(\mu-Q)}{2}\right\}-s_r\left\{\frac{[\sigma^2+(Q-\mu)^2]^{\frac{1}{2}}-(Q-\mu)}{2}\right\}$$

$$=p\left[\mu+Q-\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2}\right]-w\left[\mu+Q-\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2}\right]$$

$$-s_r\left[\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)}{2}\right]-tQ\le p\left[\mu+Q-\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2}\right]$$

holds if and only if

$$\left[Q-\frac{\sqrt{\sigma^2+(Q-\mu)^2}-(\mu-Q)}{2}\right]>0 (9)$$

From the above conditions (9), one has

$$Q+\mu>\sqrt{\sigma^2+(Q-\mu)^2} (10)$$

Solving the above equations, one can obtain $\mu > 0$. Thus, for $\mu > 0$,

$$E\left(\pi_r^{TS}\right) \le p\left[\mu + Q - \frac{\sqrt{\sigma^2 + (Q-\mu)^2} - (\mu - Q)}{2}\right] - tq = \frac{p\left[3\mu - Q - \sqrt{(Q-\mu)^2}\right]}{2} - tQ\,.$$

G. Consignment Policy (CP) –

In this case, manufacturer and retailer act as the leader and follower, respectively. Manufacturer determines the number of units to be provided to retailer as a consignment stock and offers the retailer a per-unit commission, a fixed fee and optimal consignment policy.Retailer agrees with the contract provided that it earns per-unit commission and fixed fee. The closed type equilibrium solutions are derived in this model. The revenue for the retailer comes from per unit commission and the fixed fee paid by the manufacturer. In this policy, the retailer does not carry the total inventory holding cost as the ownership of the products is retained by the manufacturer.

The operational part of the holding cost, which includes storage space, material handling, etc., is carried by the retailer. But, the financial component (opportunity cost while investing the capital, taxes, etc.) is incurred by the manufacturer.

Thus, the total cost faced by the retailer in this policy consists of the operational part of the holding cost and the goodwill loss during stockout (Fig. 3).

The expected total profit of the retailer for consignment policy is

$$E\left(\pi_r^{CP}\right) = \left\{\begin{matrix} \alpha\mu - h_r^{CP}(Q-D)^{+}A;\ D \le Q \\ \alpha Q - s_r E(D-Q)^{+}A;\ Q < D \end{matrix}\right\} (11)$$

Using (2) and (3), the expected profit of (11) can be written as

$$E\left(\pi_r^{CP}\right) = \alpha(\mu+Q) - h_r^{CP}\left\{\frac{[\sigma^2 + (Q-\mu)^2]^{\frac{1}{2}} - (\mu - Q)}{2}\right\} - s_r\left\{\frac{[\sigma^2 + (Q-\mu)^2]^{\frac{1}{2}} - (Q-\mu)}{2}\right\} + A$$

$$= \alpha(\mu+Q) - \frac{h_r^{CP} + s_r}{2}\sqrt{\sigma^2 + (Q-\mu)^2} + \frac{h_r^{CP} - s_r}{2}(\mu - Q) + A \ (12)$$

In order to maximise the profit of the retailer, one can obtain derivative of (12) with respect to *Q*, which gives,

$$\frac{\partial E\left(\pi_r^{CP}\right)}{\partial Q} - \alpha - \frac{\left(h_r^{CP} + s_r\right)(Q-\mu)}{2\sqrt{\sigma^2 + (Q-\mu)^2}} - \frac{h_r^{CP} - s_r}{2}$$

Now, equating the above equation to zero, one can obtain

$$Q_r^{CP*} - \mu + \frac{\sigma\Gamma_{CP}}{\sqrt{1 - \Gamma_{CP}^2}} \ (13)$$

where, $$\Gamma - \frac{2\alpha - \left(h_r^{CP} - s_r\right)}{h_r^{CP} + s_r}.$$

Now, at the manufacturer's end, though the entire lot is delivered to the retailer's warehouse the manufacturer keeps the ownership of the merchandise. The financial part of the holding cost is carried by the manufacturer. The selling price is fixed by the manufacturer and the retailer transfer the balance amount after deducting the per unit commission. This balance amount is the source of revenue for the manufacturer. No fund transfer is tobe taken place if the item is unsold. Besides this the manufacturer also agreed to pay a fixed fee to the retailer. The expected total profit of the manufacturer is

$$\pi_m^{CP} = \left\{\begin{matrix} (p-\alpha)\mu - cQ - tQ - h_m^{CP}E(Q-D)^{+} - A;\ D \le Q \\ (p-\alpha-c)Q - tq - s_m E(D-Q)^{+} - A;\ D > Q \end{matrix}\right\} (14)$$

Using (2) and (3), the expected profit of (14) can be written as

$$E\left(\pi_m^{CP}\right)=p(\mu+Q)-\alpha\mu-\alpha Q-cQ-tq-h_m^{CP}\left\{\frac{[\sigma^2+(Q-\mu)^2]^{\frac{1}{2}}-(\mu-Q)}{2}\right\}$$

$$-s_m\left\{\frac{[\sigma^2+(Q-\mu)^2]^{\frac{1}{2}}-(\mu-Q)}{2}\right\}-A=p(\mu+Q)-cQ-\alpha Q-\alpha\mu-tQ$$

$$-\frac{h_m^{CP}+s_m}{2}\sqrt{\sigma^2+(Q-\mu)^2}+\frac{h_m^{CP}-s_m}{2}(\mu-Q)-A \quad (15)$$

The joint total expected profit for manufacturer and retailer under consignment policy is

$$E\left(\pi_J^{CP}\right)=E\left(\pi_r^{CP}+\pi_m^{CP}\right)=E\left(\pi_r^{CP}\right)+E\left(\pi_m^{CP}\right)$$

i.e.,

$$E\left(\pi_J^{CP}\right)=\alpha(\mu+Q)-\frac{h_r^{CP}+s_r}{2}\sqrt{\sigma^2+(Q-\mu)^2}+\frac{h_r^{CP}-s_r}{2}(\mu-Q)$$

$$+A+p(\mu+Q)-cQ-\alpha Q-\alpha\mu-\frac{h_m^{CP}+s_m}{2}\sqrt{\sigma^2+(Q-\mu)^2}$$

$$+\frac{h_m^{CP}-s_m}{2}(\mu-Q)-A=(\alpha+p)(\mu+Q)-cQ-\alpha Q-\alpha\mu$$

$$-\frac{\left(h_r^{CP}+h_m^{CP}\right)+(s_r+s_m)}{2}\sqrt{\sigma^2+(Q-\mu)^2}+\frac{\left(h_r^{CP}+h_m^{CP}\right)-(s_r+s_m)}{2}(\mu-Q)\ -tQ (16)$$

In order to maximise the joint total expected profit, one can obtain the derivative of (16) with respect to Q.

$$\frac{\partial E\left(\pi_J^{CP}\right)}{\partial Q}=p-t-c-\frac{\left(h_r^{CP}+h_m^{CP}\right)+(s_r+s_m)}{2\sqrt{\sigma^2+(Q-\mu)^2}}-\frac{\left(h_r^{CP}+h_m^{CP}\right)+(s_r+s_m)}{2} \quad (17)$$

Equating the above equation to zero, one can obtain

$$Q_J^{CP*}=\mu+\frac{\sigma\lambda}{\sqrt{1-\lambda^2}},\text{ where, }\lambda=\frac{2(p-c)-t-\left[\left(h_r^{CP}+h_m^{CP}\right)-(s_r+s_m)\right]}{\left(h_r^{CP}+h_m^{CP}\right)+(s_r+s_m)} \quad (18)$$

H. Evaluation of per unit commission

In order to obtain the per-unit commission *a*, given by the manufacturer to the retailer, one can equate $Q_r^{CP*}=Q_J^{CP*}$ which gives the value of *a* as

$$a=\frac{h_r-h_s}{2}+\frac{h_r-h_s}{2(H_{CP}+S_{CP})}(2(p-c)-t-(H_{CP}+S_{CP})) \quad (19)$$

where $H_{CP}=\left(h_r^{CP}+h_m^{CP}\right)$ and $S_{CP}=(s_r+s_m)$.

I. Evaluation of the fixed fee paid by the manufacturer to the retailer

In consignment policy, retailers expect to earn at least as much as in traditional policy. This is the manufacturer's responsibility to ensure that the retailer's expected profit for *CP* reaches or exceeds the expected profit for traditional policy, i.e.,

$$\text{Max } z=E\left(\pi_m^{CP}\right)$$

subject to

$$E\left(\pi_r^{CP}\right)\geq E\left(\pi_r^{TS}\right), \quad (20)$$

i.e.,

$$A\geq p\left(\mu+Q_r^{TS}\right)-wQ_r^{TS}-\frac{h_r^{TS}+s_r}{2}\sqrt{\sigma^2+\left(Q_r^{TS}-\mu\right)^2}-\frac{h_r^{TS}-s_r}{2}\left(Q_r^{TS}-\mu\right)$$

$$-\alpha\left(\mu+Q_r^{CP}\right)+\frac{h_r^{CP}+s_r}{2}\sqrt{\sigma^2+\left(Q_r^{CP}-\mu\right)^2}+\frac{h_r^{CP}-s_r}{2}\left(Q_r^{CP}-\mu\right)-t \quad (21)$$

A point should be noted that this fixed fee is not always positive. This fee is calculated comparing the retailer's expected profit under CP and TP. The responsibility for the manufacturer is to confirm the greater profit of the retailer .in CP than that of TP. If this happens then replacing the inequality sign of (21) to equality, the fixed fee becomes negative, otherwise it is positive. A positive fixed fee is regarded as an 'admission fee' which is a allowance paid by the manufacturer to the retailer and a negative fixed fee is referred to as a 'royalty' paid by the retailer to the manufacturer (Chen and Liu 2008).

J. Retailer's royalty reduction

In CP, the manufacturer holds the ownership of product. The retailer just provides the warehouse space to hold products. Transportation is in the scope of manufacturer

In return, the retailer is offered with a per-unit commission on selling products and a fixed fee. On the other hand, a part of holding cost has also to be carried out by the retailer. An awkward situation for the retailer comes into the way while making the consignment contract with the manufacturer. This is a different decision for the retailer to agree with per-unit commission along with the fixed fee. Specially, this fixed fee is not always positive, which means the retailer has to give a royalty to the manufacturer. This may create an issue to sign a consignment contract as per unit commission is the only source of revenue for the retailer.

K. A proposed methodology to evaluate the fixed fee

This proposed method can reduce the royalty which has to be paid by the retailer to the manufacturer in case of negative fixed fee. One can convert the inequality sign of (21) to equality, which shows

$$A = E\left(\pi_r^{TS}\right) - E\left(\pi_r^{CP}\right)$$

The value of *A* will be negative if

$$E\left(\pi_r^{CP}\right) > E\left(\pi_r^{TS}\right) \quad (22)$$

Now, one can obtain the ratio

$$r - \left|\frac{E\left(\pi_r^{TS}\right)}{E\left(\pi_r^{CP}\right)}\right| \quad (23)$$

If the expression in (23) holds, then $r < 1$ and

$$A_n - rA - \left|\frac{E\left(\pi_r^{TS}\right)}{E\left(\pi_r^{CP}\right)}\right| A < A \quad (24)$$

Thus, the royalty, which is to be given by the retailer, can be reduced in this way. The expected profit for the retailer and the manufacturer for this new fixed cost will be

$$E\left(\pi_m^{CP}\right) = \alpha\left(\mu + Q_r^{CP}\right) - h_r^{CP}\left\{\frac{\left[\sigma^2 - \left(Q_r^{CP} - \mu\right)^2\right]^{\frac{1}{2}} - \left(\mu - Q_r^{CP}\right)}{2}\right\}$$

$$-s_r\left\{\frac{\left[\sigma^2 - \left(Q_r^{CP} - \mu\right)^2\right]^{\frac{1}{2}} - \left(Q_r^{CP} - \mu\right)}{2}\right\} - A_n - p\left(\mu - Q_r^{CP}\right) - cQ_r^{CP}$$

$$-\alpha Q_r^{CP} - \alpha\mu - \frac{h_m^{CP} + s_m}{2}\sqrt{\sigma^2 + \left(Q_r^{CP} - \mu\right)^2} - \frac{h_m^{CP} - s_m}{2}\left(\mu - Q_r^{CP}\right) - A_n - t \quad (26)$$

respectively.

IV. Solution Procedure

To evaluate the fixed fee using the proposed methodology, an algorithm is needed obtain the numerical solutions. The following

A. Solution algorithm

Step-1: Input all parametric values μ, σ,*p*, t,*w*, *c*, s_r^{TS}, s_r^{CP}, s_m^{CP}, h_r^{TS}, h_r^{CP}, , and h_m^{CP}.

Step-2: Convert all fuzzy values into crisp values using Yager's approximation

Step-3: Evaluate the order quantity for the retailer h_r^{TS} from (6).

Step-4: Utilise the value of Q_r^{TS} to obtain the retailer's total expected profit $E\left(\pi_r^{TS}\right)$ from (4) and themanufacturer's total expected profit $E\left(\pi_m^{TS}\right)$ from (7).

Step-5: Evaluate the value of per unit commission a for the retailer using (19).

Step-6: Obtain the value of the order quantity in consignment policy Q_r^{CP} for the retailer from (13).

Step-7: Evaluate the fixed cost *A*from (21).

Step-8: If *A*> 0 then evaluate the expected profit of the retailer $E\left(\pi_r^{CP}\right)$ and the manufacturer $E\left(\pi_m^{TS}\right)$ from (12) and (15) respectively.

Step-9: If *A*< 0 then execute the following steps.

Step-9a: Evaluate $A_n$from (24).

Step-9b: Utilise the value of A_n to evaluate the new expected profit for the retailer and the manufacturer i.e. $E\left(\pi_m^{CP}\right)$ and $E\left(\pi_{mn}^{CP}\right)$respectively, from (25) and (26).

A. Numerical experiment –

Example-1:

The following parameter values for traditional policy are used as follows: μ = 100, σ = 200, *p* = (28,30,32)/unit, *w* = 25/unit, *c* = 19/unit and s_r = 20/unit.t =rs 0.1 /unit Now using the solution algorithm, the optimal solutions for the traditional system model are summarised in Table-1.

Example-2:

The same parametric values are as used in Example-1. Example-2 is used for consignment policy. The inventory holding cost is divided into both of the parties. The optimal solutions for consignment policy are summarized in Table-2. This solution is obtained for general consignment policy. The optimal solutions for the proposed method to evaluate the royalty are given in Table-3.

Table-1 Optimal solutions for traditional system

h_r	Q_r^{TS}	$E\left(\pi_r^{TS}\right)$	$E\left(\pi_m^{TS}\right)$	Joint profit
0.62c[a]	232.01	852.25	1408.40	2265.50
0.71c	206.67	510.00	1214.00	1700.20

[a]0.62c refers to 62 per cent of c (manufacturing cost).

Table-2 Optimal solutions for consignment policy

(h_r, h_m)	(s_r, s_m)	α	Q_r^{CP}	$E\left(\pi_r^{CP}\right)$	$E\left(\pi_m^{TS}\right)$	Joint profit	A
(0.20c, 0.45c)	(0, 21)	3.84	632.32	845.25	2100.19	2970.44	+210.44
(0.25c, 0.48c)	(0, 21)	4.25	392.35	520.00	1645.27	2180.27	–125.13*

*'+' indicates the amount is transferred from manufacturer to retailer and conversely, '–' is for retailer to manufacturer

Table-3 Optimal solutions for the proposed method in consignment policy

(h_r, h_m)	(s_r, s_m)	α	Q_r^{CP}	$E\left(\pi_r^{CP}\right)$	$E\left(\pi_m^{TS}\right)$	**Joint profit**	*A*
(0.20c, 0.48c)	(0, 21)	4.25	392.35	523.14	1643.13	2152.27	–99.20*

*'+' indicates the amount is transferred from manufacturer to retailer and conversely,
'–' is for retailer to manufacturer

B. Price-sensitivity on demand –

Sensitivity analysis is performed for the main key parameters, holding costs in consignment policy. The percentage of variation isconsidered from –10 to –10% for each cost components, financial part and operational part of the holding cost, respectively. The sensitivity of both manufacturer's and retailer's profit is analysed as well as the joint total profit of the supply chain. The results are given in Table-4.

Profits of retailer, manufacturer and joint profit are less sensitive with operational part of holding cost than financial part.

- Retailer's profit is more sensitive, when operational part of holding cost increases positively, but less sensitive when decreases.

Table-4 Sensitivity analysis for bolding cost components

Parameters	**Changes (in %)**	**Retailer's profit**	**Manufacturer's profit**	**Joint profit**
h_r^{CP}				
	–10	11.14	6.65	17.79
	–5	4.56	3.53	8.09
	+5	–2.32	–3.89	–6.703
	+10	–4.12	–8.08	–12.20
h_m^{CP}				
	–10	24.71	11.25	35.96
	–5	11.46	5.49	16.94
	+5	–9.48	–5.45	–14.93
	–10	–16.140	–11.06	–27.86

From the above it can be felt.

- A consignment contract saves fund in favor of retailer as the financial and operational part of holding cost is divided into retailer and manufacturer, respectively.
- Demand is considered as random but free of any specific distribution An effective procedure is established in reduce retailer's royalty to strengthen the negotiation between manufacturer and retailer in achieving a consignment contract

V. CONCLUSIONS

In this paper, a comparison between the traditional policy and the consignment policy for ainventory problem with the manufacturer and the retailer .The demand distribution is completely unknown except a known mean and standard deviation depending on which the entire model explained including fuzzy price and transportation cost . The optimal decisions were obtained for both traditional and consignment policy It was observed that the joint profit for the consignment policy is greater than that of the traditional policy.Using the proposed method, the royalty for the retailer was reduced without affecting die joint total expected profit for both parties. The price-sensitivity or demand or retail price markdown was performed. The model can be extended with delay-in-payments and the concept of fixed life products .it can also be extended for multiple products and multi period environment.

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