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ENHANCED PERFORMANCE WITH PID CONTROLLERS AND LEAD/LAG COMPENSATORS FOR UNSTABLE PROCESSES WITH TIME DELAYS

J.Udaya Sekhar[1], Dr. B. Sarath Babu [1]

Abstract- Using the direct synthesis method, a proportional-integral-derivative (PID) controller in series with a lead- lag filter is designed for control of the open-loop unstable processes with time delay. Different types of approximations (Taylors 1st and 2nd order, Pade's 1st, 2nd and ½ orders) for the time delay term are considered and obtained different controller structures of PID. A comparative study of the performance of the designed controllers has been carried out on various unstable processes. Set-point weighting is considered to reduce the overshoot. The proposed scheme consists of only one tuning parameter, and systematic guidelines are provided for selection of the tuning parameter based on the peak value of the sensitivity function. Robustness analysis has been carried out based on sensitivity and complementary sensitivity functions. Nominal and robust control performances are achieved with the proposed method and improved closed loop performances are obtained when compared to the recently reported methods in the literature.

Key Words: unstable process, time delay, direct synthesis method, sensitivity function.

I. INTRODUCTION

Open-loop unstable processes are much more difficult to control than that of the stable processes. The difficulty increases when the process contains a time delay. Extensive information on the physical significance of the unstable systems in the context of aero planes has been given by Stein [1]. Desired closed-loop performance cannot be achieved with the conventional proportional-integral-derivative (PID) controllers for any adjustable parameters of the PID parameters [2]. Time delays occur frequently in process control problems, because of the distance velocity lags, recycle loops, and composition analysis loops, or in the approximation of higher-order systems with a lower-order system with a time delay. The performance specifications that are usually achieved for stable systems are difficult to achieve for unstable systems. The performance of the closed loop response for such processes exhibits large overshoots and settling times. The dynamics of many processes can be represented by first or second order processes plus time delay. For unstable first order plus time delay (UFOPTD) processes, the existence of a right-half plane pole and time delay makes it difficult to stabilize the system, particularly with the conventional proportional-integral/proportional-integral derivative (PI/PID) controllers. The controller design methods for the UFOPTD processes have been addressed by many researchers [3-8]. Many chemical and biological systems exist whose dynamics also show second-order behavior. These types of systems can be modeled as open-loop unstable second-order plus time delay (USOPTD) models. Controlling these types of processes is difficult, and this difficulty increases when the USOPTD model contains a positive or negative zero. Controller design methods for unstable second-order processes is described by Huang and Chen [9], Lee et al. [10] , Yang et al. [11] , Wang and Cai [12], Kwak et al. [2], Tan et al.[13], Lu et al. [14], and Liu et al. [15], Wang and Hwang [16]. Some of the recently

[1] *Department of Chemical Engineering , S. V. University College of Engineering, S.V. University, Tirupati, India*

reported methods use a two-degrees-of-freedom control structure with a greater number of controllers and also with great complexity in the design of controllers. Recently, Rao and Chidambaram [17] have proposed a controller design method for USOPTD processes with two RHP poles and a zero.

Direct synthesis method is a well know technique for design of controllers and the main advantage of this method is that the desired output behavior of the closed loop can be specified as a trajectory model based on the process to design the required form of the controller [18]. Many design methods have been proposed in the literature based on direct synthesis method for unstable processes. Approximation of the time delay parameter places major role to obtain the controller structure in direct synthesis method. Many of the existing design methods make use of first order Pade's approximation for time delay to derive the controller parameters. In this work, an attempt is made by considering different type of approximations which includes Taylor's 1st and 2nd order, Pade's 1st, 2nd and ½ order for the time delay term to obtain improved performance. For clear illustration, the paper is organized as follows. Theoretical developments and design is given in section 2 followed by stability and robustness studies in section 3. Simulation studies are explained in section 4 and conclusions are presented in section 5.

II. THEORETICAL DEVELOPMENTS

The closed-loop control structure is shown in Figure 1, where $G_p(s)$ is the process transfer function and $G_c(s)$ is the transfer function of the controller. The closed-loop transfer function for the set-point changes is given by

$$\frac{y(s)}{y_r(s)} = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \tag{1}$$

From eq 1, using the direct synthesis method, the controller expression is obtained as

$$G_c(s) = \frac{1}{G_p}\frac{(y/y_r)_d}{[1-(y/y_r)_d]} \tag{2}$$

Here, $(y/y_r)_d$ is the desired closed-loop transfer function for a set-point change. The desired closed-loop transfer function should be assumed such that the resulting controller is realizable

A. Controller design for UFOPTD processes –

Typical UFOPTD processes exist in most of the chemical and biological systems can be represented by the following transfer function model.

$$G_p(s) = \frac{K_p e^{-\theta s}}{\tau s-1} \tag{3}$$

For direct synthesis method, the desired closed-loop transfer function is assumed as

$$\frac{y(s)}{y_r(s)} = \frac{(\eta s+1)e^{-\theta s}}{(\lambda s+1)^2} \tag{4}$$

From eq 3 & 4, the controller is obtained as

$$G_c(s) = \frac{(\eta s+1)}{K_p}\left[\frac{(\tau s-1)}{(\lambda s+1)^2-(\eta s+1)e^{-\theta s}}\right] \tag{5}$$

Based on the different types of approximation used for the time delay term in eq 5, the controller expressions are obtained. Here, Pade's 1st, 2nd and ½ order approximations, Taylor's 1st and 2nd order for time delay term are considered.

Case-1: Pade's ½ order

Pade's ½ order approximation is defined by the following equation

$$e^{-\theta s} = \frac{6-2\theta s}{6+4\theta s+\theta^2 s^2} \tag{6}$$

With this approximation, the controller (eq.5) can be approximated in the form of

$$G_c = \frac{(\eta s+1)(\tau s-1)(6+4\theta s+\theta^2 s^2)}{K_p h s(x_1 s^3+x_2 s^2+x_3 s+1)} \tag{7}$$

$where\, h = 12\lambda + 6\theta - 6\eta, x_1 = \lambda^2\theta^2/h, x_2 = (4\lambda^2\theta + 2\lambda\theta^2)/h, x_3 = (6\lambda^2 + 8\lambda\theta + \theta^2 + 2\theta\eta)/h$

The denominator term in eq 7, $x_1s^3 + x_2s^2 + x_3s + 1$, can be factorized as

$$x_1s^3 + x_2s^2 + x_3s + 1 = (1-\tau s)(1+\beta_1 s+\beta_2 s^2) \tag{8}$$

Upon equating the corresponding coefficients on both sides of eq 8, we get the coefficients η, β_1, and β_2 as

$$\beta_1 = \tau + [(6\lambda^2 + 8\lambda\theta + \theta^2 + 2\theta\eta)/h],\ \beta_2 = (-\lambda^2\theta^2)/\tau h$$

$$\eta = \frac{\lambda^2\left(4\theta + 6\tau + \theta^2/\tau\right) + \lambda\left(2\theta^2 + 8\theta\tau + 12\tau^2\right) + 6\theta\tau^2 + \tau\theta^2}{6\tau^2 - 2\theta\tau}$$

Thus, the final controller $Gc(s)$ form is obtained as

$$G_c = k_c\left(1 + \frac{1}{\tau_i s}\right)\left(\frac{1 + \alpha_1 s + \alpha_2 s^2}{1 + \beta_1 s + \beta_2 s^2}\right) \tag{9}$$

Where $K_c = -6\eta/(k_p h)$, $\tau_I = \eta$, $\alpha_1 = 2\theta/3, \alpha_2 = \theta^2/6$ and λ is the tuning parameter.

Case-2: Pade's 1st order

Pade's 1st order approximation is defined by the following equation

$$e^{-\theta s} = \frac{6 - 2\theta s}{6 + 4\theta s + \theta^2 s^2} \tag{10}$$

With this, the controller is obtained as

$$G_c = \frac{(\eta s + 1)(\tau s - 1)(1 + 0.5\theta s)}{K_p hs\left(x_1 s^2 + x_2 s + 1\right)} \tag{11}$$

Where $h = 2\lambda + \theta - \eta, x_1 = 0.5\lambda^2\theta/h, x_2 = (\lambda^2 + \lambda\theta + 0.5\theta\eta)/h$

The denominator term in eq 11, $x_1s^2 + x_2s + 1$, can be factorized as

$$x_1s^2 + x_2s + 1 = (1-\tau s)(1+\beta s) \tag{12}$$

Upon equating the corresponding coefficients on both sides of above eq 12, we get the coefficients η, β and α as

$$\beta = -0.5\lambda^2\theta/\tau h, \alpha = 0.5\theta, \eta = [\lambda^2(1 + 0.5\theta/\tau) + \lambda(\theta + 2\tau) + \theta\tau]/(\tau - 0.5\theta)$$

With that the final controller $Gc(s)$ form is obtained as

$$G_c = k_c\left(1 + \frac{1}{\tau_i s}\right)\left(\frac{1 + \alpha s}{1 + \beta s}\right) \tag{13}$$

Where $K_c = -\eta/(k_p h)$, $\tau_I = \eta$ in which λ is the tuning parameter.

Case-3: Pade's 2nd order

Pade's 2nd order approximation is defined by the following equation

$$e^{-\theta s} = \frac{12 - 6\theta s + \theta^2 s^2}{12 + 6\theta s + \theta^2 s^2} \tag{14}$$

With this the controller is obtained as

$$G_c = \frac{(\eta s + 1)(\tau s - 1)\left(12 + 6\theta s + \theta^2 s^2\right)}{K_p hs\left(x_1 s^3 + x_2 s^2 + x_3 s + 1\right)} \tag{15}$$

$$where \quad h=(2\lambda+\theta-\eta), x_1=\lambda^2\theta^2/12h, \quad x_2=[(\lambda^2\theta/2)+(\lambda\theta^2/6)-(\theta^2\eta/12)]/h$$
$$x_3=[\lambda^2+\lambda\theta+0.5\theta\eta]/h$$

The denominator term in eq 15, $x_1s^3 + x_2s^2 + x_3s + 1$, can be factorized as

$$x_1s^3 + x_2s^2 + x_3s + 1 = (1-\tau s)(1+\beta_1 s + \beta_2 s^2) \tag{16}$$

Upon equating the corresponding coefficients on both sides of eq 16, we get the coefficients η, β_2, and β_1 as

$$\beta_1=\tau+(\lambda^2+\lambda\theta+0.5\theta\eta)/h, \beta_2=-\lambda^2\theta^2/12\tau h$$
$$\eta=[\lambda^2\left(6\theta+12\tau+(\theta^2/\tau)\right)+\lambda\left(2\theta^2+12\theta\tau+24\tau^2\right)+12\theta\tau^2]/(12\tau^2+\theta^2-6\theta\tau)$$

Thus, the final controller $Gc(s)$ form is obtained as

$$G_c=k_c\left(1+\frac{1}{\tau_i s}\right)\left(\frac{1+\alpha_1 s+\alpha_2 s^2}{1+\beta_1 s+\beta_2 s^2}\right) \tag{17}$$

Where $K_c = -\eta/(k_p h)$, $\tau_I = \eta$, $\alpha_1 = \theta/2$, $\alpha_2 = \theta^2/12$ and λ is the tuning parameter.

Case- 4: Taylor's 1st order

Taylor's 1st order approximation is defined by the following equation

$$e^{-\theta s}=1-\theta s \tag{18}$$

With this approximation, the controller (eq.5) can be approximated in the form of

$$G_c=\frac{(\eta s+1)(\tau s-1)}{K_p hs(xs+1)} \tag{19}$$

$$where \quad h=2\lambda+\theta-\eta, x=\lambda^2+\eta\theta/h$$

By equating the denominator term in eq 19 as xs +1= (1-τs), the coefficient η is obtained as

$$\eta=\frac{\lambda^2+2\lambda\tau+\theta\tau}{\tau-\theta}$$

With that the final controller $Gc(s)$ form is obtained as

$$G_c=k_c\left(1+\frac{1}{\tau_i s}\right) \tag{20}$$

Where $K_c = -\eta/(k_p h)$, $\tau_I = \eta$, and λ is the tuning parameter.

Case-5: Taylor's 2nd order

Taylor's 2nd order approximation is given by the following equation

$$e^{-\theta s}=1-\theta s+0.5\theta^2 s^2 \tag{21}$$

With this, the controller is obtained as

$$G_c=\frac{(\eta s+1)(\tau s-1)}{K_p hs(x_1 s^2+x_2 s+1)} \tag{22}$$

$$where \quad h=2\lambda+\theta-\eta, \; x_1=-0.5\eta\theta^2/h, \; x_2=(\lambda^2+\eta\theta-0.5\theta^2)/h$$

The denominator term in eq 22, $x_1s^2 + x_2s + 1$, can be factorized as

$$x_1s^2 + x_2s + 1 = (1-\tau s)(1+\alpha s) \tag{23}$$

Upon equating the corresponding coefficients on both sides of above eq 23, we get the coefficients η and α as

$$\alpha = 0.5\eta\theta^2 / \tau h, \eta = [2\lambda^2\tau + 4\tau^2\lambda + \theta\tau(2\tau - \theta)] / (\theta^2 + 2\tau^2 - 2\theta\tau)$$

With that, the final controller $Gc(s)$ form is obtained as

$$G_c = k_c\left(1 + \frac{1}{\tau_i s}\right)\left(\frac{1}{1 + \alpha s}\right) \tag{24}$$

Where $K_c = -\eta/(k_p h)$, $\tau_I = \eta$ in which λ is the tuning parameter.

All the controllers structures corresponding to the time delay approximations is given in Table 1 for clear illustration.

B. Controller design for USOPTD processes

The typical USOPTD processes exist in most of the chemical and biological systems can be represented by any of the following transfer function models:

$$G_p(s) = \frac{k_p e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s - 1)} \tag{25}$$

$$G_p(s) = \frac{k_p e^{-\theta s}}{(\tau_1 s - 1)(\tau_2 s - 1)} \tag{26}$$

$$G_p(s) = \frac{k_p e^{-\theta s}}{s(\tau s - 1)} \tag{27}$$

Of all the processes, the one that is difficult to control is the USOPTD process with two RHP poles. For generalization, the process considered for the design of the controller is

$$G_p(s) = \frac{k_p e^{-\theta s}}{a_1 s^2 + a_2 s + 1} \tag{28}$$

Where $a1 > 0$, $a2 < 0$, and the open-loop RHP poles of $Gp(s)$ may be real or complex.

In eq. 2, the desired closed-loop transfer function should be assumed such that the resulting controller is realizable according to eq. 28. The desired closed-loop transfer function is assumed as

$$\frac{y(s)}{y_r(s)} = \frac{(\eta_2 s^2 + \eta_1 s + 1)e^{-\theta s}}{(\lambda s + 1)^3} \tag{29}$$

From eq 2, the controller is obtained as

$$G_c(s) = \frac{a_1 s^2 + a_2 s + 1}{k_p}\left[\frac{\eta_2 s^2 + \eta_1 s + 1}{(\lambda s + 1)^3 - (\eta_2 s^2 + \eta_1 s + 1)e^{-\theta s}}\right] \tag{30}$$

Different types of approximations are considered for the time delay term in eq 30 and the controller expressions are obtained.

Case-1: Pade's ½ order

Using eq 6, the controller (eq.30) can be approximated in the form of

$$G_c(s) = \frac{(a_1 s^2 + a_2 s + 1)(\eta_2 s^2 + \eta_1 s + 1)(6 + 4\theta s + \theta^2 s^2)}{k_p h s(x_1 s^4 + x_2 s^3 + x_3 s^2 + x_4 s + 1)} \tag{31}$$

Where $h = 18\lambda + 6\theta - 6\eta, x_1 = \lambda^3\theta^2 / h, x_2 = (4\lambda^3\theta + 3\lambda^2\theta^2) / h, x_3 = (6\lambda^3 + 12\lambda^2\theta + 3\lambda\theta^2 + 2\theta\eta_2) / h$

$$x_4 = (18\lambda^2 + 12\lambda\theta + \theta^2 + 2\theta\eta_1 - 6\eta_2)/h$$

The denominator term in eq 31, $x_1s^4 + x_2s^3 + x_3s^2 + x_4s+1$, can be factorized as

$$x_1s^4 + x_2s^3 + x_3s^2+x_4s+1= (a_1s^2+a_2s+1)(1+\beta_1s+\beta_2s^2) \tag{32}$$

Upon equating the corresponding coefficients on both sides of eq 32, we get

$$a_1\beta_2=x_1,\ a_1\beta_1+a_2\beta_2=x_2,\ a_1+a_2\beta_1+\beta_2=x_3,\ \beta_1+a_2=x_4$$

From the above-mentioned relations, the coefficients n_2, n_1, β_2 and β_1 are obtained as

$$\beta_1 = \left(18\lambda^2 + 12\lambda\theta + \theta^2 + 2\theta\eta_1 - 6\eta_2/h\right) - a_2,\ \beta_2 = \lambda^3\theta^2/a_1h$$

$$\eta_1 = \frac{y_2z_2 - z_1y_4}{y_2y_3 - y_1y_4} \qquad \eta_2 = \frac{y_3z_1 - z_2y_1}{y_2y_3 - y_1y_4}$$

Where

$$y_1 = 2a_1\theta + 6a_1a_2,\ y_2 = -6a_1,\ y_3 = 2a_2\theta + 6(a_2^2 - a_1),\ y_4 = -2\theta - 6a_2$$

$$z_1 = \lambda^3\left(4\theta - a_2\theta^2/a_1\right) + \lambda^2\left(3\theta^2 - 18a_1\right) + \lambda\left(18a_1a_2 - 12a_1\theta\right) + 6a_1a_2\theta - a_1\theta^2$$

$$z_2 = \lambda^3\left(6 - \theta^2/a_1\right) + \lambda^2\left(12\theta - 18a_2\right) + \lambda\left(3\theta^2 + 18(a_2^2 - a_1) - 12a_2\theta\right) + 6\theta(a_2^2 - a_1) - a_2\theta^2$$

With that the final controller $Gc(s)$ form is obtained as

$$G_c = k_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right)\left(\frac{1 + \alpha_1 s + \alpha_2 s^2}{1 + \beta_1 s + \beta_2 s^2}\right) \tag{33}$$

Where $K_c = 6\eta_1/(k_p h)$, $\tau_I = \eta_1$, $\tau_D = \eta_2/\eta_1$, $\alpha_1 = 2\theta/3, \alpha_2 = \theta^2/6$ and λ is the tuning parameter.

Case-2: Pade's 1st order

From eq 10, the controller (eq.30) is obtained as

$$G_c(s) = \frac{\left(a_1s^2 + a_2s + 1\right)\left(\eta_2s^2 + \eta_1s + 1\right)\left(1 + 0.5\theta s\right)}{k_p hs\left(x_1s^3 + x_2s^2 + x_3s + 1\right)} \tag{34}$$

$$where\ h = 3\lambda + \theta - \eta_1, x_1 = 0.5\lambda^3\theta/h, x_2 = (\lambda^3 + 1.5\lambda^2\theta + 0.5\eta_2\theta)/h, x_3 = (3\lambda^2 + 1.5\lambda\theta + 0.5\theta\eta_1 - \eta_2)/h$$

The denominator term in eq 34, $x_1s^3 + x_2s^2 + x_3s + 1$, can be factorized as

$$x_1s^3 + x_2s^2 + x_3s + 1 = (a_1s^2+a_2s+1)(1+\beta s) \tag{35}$$

Upon equating the corresponding coefficients on both sides of eq 35, we get

$$a_1\beta=x_1,\ a_1+a_2\beta= x_2,\ \beta+a_2= x_3$$

From the above-mentioned relations, the coefficients n_2, n_1 and β are obtained as

$$\beta = 0.5\lambda^3\theta/a_1h \qquad \eta_1 = \frac{y_2z_2 - z_1y_4}{y_2y_3 - y_1y_4} \qquad \eta_2 = \frac{y_3z_1 - z_2y_1}{y_2y_3 - y_1y_4}$$

Where

$$y_1 = -a_1^2,\ y_2 = -0.5a_1\theta,\ y_3 = -a_1a_2 - 0.5a_1\theta,\ y_4 = a_1$$

$$z_1 = \lambda^3\left(a_1 - 0.5a_2\theta\right) + 1.5\lambda^2a_1\theta - 3a_1^2\lambda - a_1^2\theta$$

$$z_2 = -0.5\lambda^3\theta + 3\lambda^2a_1 + \lambda\left(1.5a_1\theta - 3a_1a_2\right) - a_1a_2\theta$$

The final controller $Gc(s)$ form is obtained as

$$G_c = k_c\left(1+\frac{1}{\tau_i s}+\tau_d s\right)\left(\frac{1+\alpha s}{1+\beta s}\right) \tag{36}$$

Where $K_c = \eta_1/(k_p h)$, $\tau_I = \eta_1$, $\tau_D = \eta_2/\eta_1$, $\alpha = 0.5\theta$ and λ is the tuning parameter.

Case-3: Pade's 2nd order

Here, using eq 14, the controller (eq.30) is approximated as

$$G_c(s) = \frac{\left(a_1 s^2 + a_2 s + 1\right)\left(\eta_2 s^2 + \eta_1 s + 1\right)\left(1+\theta s/2 + \theta^2 s^2/12\right)}{k_p hs\left(x_1 s^4 + x_2 s^3 + x_3 s^2 + x_4 s + 1\right)} \tag{37}$$

$$where\ \ h = 3\lambda + \theta - \eta_1, x_1 = \lambda^3\theta^2/12h, x_2 = (0.5\lambda^3\theta + 0.25\lambda^2\theta^2 - \eta_2\theta^2/12)/h$$

$$x_3 = \left(\lambda^3 + 1.5\lambda^2\theta + 0.25\lambda\theta^2 + 0.5\theta\eta_2 - \eta_1\theta^2/12\right)/h, x_4 = (3\lambda^2 + 1.5\lambda\theta + 0.5\theta\eta_1 - \eta_2)/h$$

The denominator term in eq 37, $x_1s^4 + x_2s^3 + x_3s^2 + x_4s+1$, can be factorized as

$$x_1s^4 + x_2s^3 + x_3s^2 + x_4s+1 = (a_1s^2+a_2s+1)(1+\beta_1 s+\beta_2 s^2) \tag{38}$$

Upon equating the corresponding coefficients on both sides of eq 38, we get

$$a_1\beta_2=x_1,\ a_1\beta_1+a_2\beta_2=x_2,\ a_1+a_2\beta_1+\beta_2=x_3,\ \beta_1+a_2=x_4$$

From the above-mentioned relations, the coefficients n_2, n_1, β_2 and β_1 are obtained as

$$\beta_1 = \left(3\lambda^2 + 1.5\lambda\theta + 0.5\theta\eta_1 - \eta_2/h\right) - a_2, \beta_2 = \lambda^3\theta^2/12a_1 h$$

$$\eta_1 = \frac{y_2 z_2 - z_1 y_4}{y_2 y_3 - y_1 y_4} \qquad \eta_2 = \frac{y_3 z_1 - z_2 y_1}{y_2 y_3 - y_1 y_4}$$

Where

$$y_1 = 6a_1\theta + 12a_1a_2, y_2 = \theta^2 - 12a_1, y_3 = \theta^2 + 6a_2\theta + 12(a_2^2 - a_1), y_4 = -6\theta - 12a_2$$

$$z_1 = \lambda^3\left(6\theta - a_2\theta^2/a_1\right) + \lambda^2\left(3\theta^2 - 36a_1\right) + \lambda\left(36a_1a_2 - 18a_1\theta\right) + 12a_1a_2\theta$$

$$z_2 = \lambda^3\left(12 - \theta^2/a_1\right) + \lambda^2\left(18\theta - 36a_2\right) + \lambda\left(3\theta^2 + 36(a_2^2 - a_1) - 18a_2\theta\right) + 12\theta(a_2^2 - a_1)$$

Thus, the final controller $Gc(s)$ form is obtained as

$$G_c = k_c\left(1+\frac{1}{\tau_i s}+\tau_d s\right)\left(\frac{1+\alpha_1 s+\alpha_2 s^2}{1+\beta_1 s+\beta_2 s^2}\right) \tag{39}$$

Case-4: Taylor's 1st order

Using eq. 18, the controller is obtained as

$$G_c(s) = \frac{\left(a_1 s^2 + a_2 s + 1\right)\left(\eta_2 s^2 + \eta_1 s + 1\right)}{k_p hs\left(x_1 s^2 + x_2 s + 1\right)} \tag{40}$$

Where $h = 3\lambda + \theta - \eta_1, x_1 = \lambda^3 + \eta_2\theta/h, x_2 = (3\lambda^2 + \eta_1\theta - \eta_2)/h$

The denominator term in eq 41, $x_1s^2 + x_2s + 1$, can be equated to

$$x_1s^2 + x_2s + 1 = a_1s^2 + a_2s + 1 \tag{41}$$

Upon equating the corresponding coefficients on both sides of eq 41, we get

$$a_1=x_1,\ a_2=x_2$$

From the above-mentioned relations, the coefficients n_2 and n_1 are obtained as

$$\eta_1 = \frac{(a_1 + a_2\theta)(3\lambda + \theta) - \lambda^3 - 3\lambda^2\theta}{\theta^2 + a_2\theta + a_1}, \eta_2 = \frac{a_1(3\lambda + \theta) - a_1\eta_1 - \lambda^3}{\theta}$$

Thus, the final controller $Gc(s)$ form is obtained as

$$G_c = k_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right) \tag{42}$$

Where $K_c = \eta_1/(k_p h)$, $\tau_I = \eta_1$, $\tau_D = \eta_2/\eta_1$ and λ is the tuning parameter.

Case-5: Taylor's 2nd order

From eq 21, the controller is approximated in the form of

$$G_c(s) = \frac{(a_1 s^2 + a_2 s + 1)(\eta_2 s^2 + \eta_1 s + 1)}{k_p hs(x_1 s^3 + x_2 s^2 + x_3 s + 1)} \tag{43}$$

$$where\ h = 3\lambda + \theta - \eta_1, x_1 = 0.5\eta_2\theta^2/h, x_2 = (\lambda^3 + \eta_2\theta - 0.5\eta_1\theta^2)/h, x_3 = (3\lambda^2 + \eta_1\theta - 0.5\theta^2 - \eta_2)/h$$

The denominator term in eq 43, $x_1s^3 + x_2s^2 + x_3s + 1$, can be factorized as

$$x_1s^3 + x_2s^2 + x_3s + 1 = (a_1s^2 + a_2s + 1)(1 + \alpha s) \tag{44}$$

Upon equating the corresponding coefficients on both sides of eq 44, we get

$$a_1\alpha = x_1,\ a_1 + a_2\alpha = x_2,\ \alpha + a_2 = x_3$$

From the above-mentioned relations, the coefficients n_2, n_1, and α are obtained as

$$\alpha = -0.5\eta_2\theta^2/a_1 h \quad \eta_1 = \frac{y_2 z_2 - z_1 y_4}{y_2 y_3 - y_1 y_4} \quad \eta_2 = \frac{y_3 z_1 - z_2 y_1}{y_2 y_3 - y_1 y_4}$$

Where $y_1 = -2a_1^2 + a_1\theta, y_2 = -a_2\theta^2 - 2a_1\theta, y_3 = -2a_1a_2 - 2a_1\theta, y_4 = 2a_1 - \theta^2$

$$z_1 = 2a_1\lambda^3 - 2a_1^2(3\lambda + \theta) \quad z_2 = a_1(6\lambda^2 - \theta^2) - 2a_1a_2(3\lambda + \theta)$$

Thus, the final controller $Gc(s)$ form is obtained as

$$G_c = k_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right)\left(\frac{1}{1 + \alpha s}\right) \tag{45}$$

Where $K_c = \eta_1/(k_p h)$, $\tau_I = \eta_1$, $\tau_D = \eta_2/\eta_1$ and λ is the tuning parameter.

Because there exists always a tradeoff between the nominal performance and robust performance, the tuning parameter (λ) must be tuned according to the desired choice. The derived controller structures are given in Table 2.

C. *Set-Point Weighting*

The PID controller designed based on the direct synthesis method usually introduces a zero in the closed-loop transfer function. This closed loop transfer function zero introduces an overshoot for the servo response [19, 20]. To reduce the undesirable overshoot, set-point weighting is suggested [21]. Hence, in the present work, set-point weighting is considered to reduce the overshoot. With the set-point weighting, the PI controller can be implemented in the form of

$$u(t) = k_c\left[(\varepsilon y_r - y) + \left(\frac{1}{\tau_i}\right)\int e\ dt\right] \tag{46a}$$

With the set-point weighting, the PID controller can be implemented in the form of

$$u(t) = k_c\left[(\varepsilon y_r - y) + \left(\frac{1}{\tau_i}\right)\int e\, dt + \tau_d \frac{de}{dt}\right] \tag{46b}$$

Where ε is the set point weighting parameter and its range is $0 < \varepsilon < 1$. The set-point weighting parameter should be selected such that the closed-loop response should give less overshoot and settling time. Based on many simulation studies on different types of UFOPTD and USOPTD processes, ε is recommended as 0.1.

III. STABILITY AND ROBUSTNESS

For any closed-loop control system, it is necessary to analyze the stability and robustness in the presence of model uncertainties. The types of uncertainties considered here are the parametric uncertainties such as uncertainty in process gain, time constant, and time delay. The closed loop system (Figure 1) is robustly stable if and only if [22]

$$\left\| l_m(j\omega)\, T(j\omega) \right\| < 1 \qquad \forall\omega(-\infty,\infty) \tag{47}$$

Where $T(s{=}j\omega)$ is the complementary sensitivity function and $l_m(s{=}j\omega)$ is the bound on the process multiplicative uncertainty. The process uncertainty can be represented as

$$l_m(j\omega) = \left|\frac{G_p(j\omega) - G_m(j\omega)}{G_m(j\omega)}\right| \tag{48}$$

Where $G_m\,(j\omega)$ is the model of the unstable process.
For UFOPTD processes, If uncertainty exists in the time delay, then the tuning parameter should be selected such that

$$\left\| T(j\omega) \right\|_\infty < \frac{1}{\left| e^{-\Delta\theta s} - 1 \right|} \tag{49}$$

If the uncertainty exists in the gain, then the tuning parameter should be selected in such a way that

$$\left\| T(j\omega) \right\|_\infty < \frac{1}{\left|\Delta k\right| / k_p} \tag{50}$$

Similarly, if the uncertainty exists in τ, then by similar analysis, the value of λ should satisfy the constraint

$$\left\| T(j\omega) \right\|_\infty < \left|\frac{\tau s - 1}{\Delta\tau s}\right| \tag{51}$$

Also, to ensure robust closed loop performance, the constraints to be followed by the sensitivity and complementary sensitivity functions are [22]

$$\left\| l_m(j\omega)T(j\omega) + w_m(j\omega)S(j\omega)) \right\| < 1 \tag{52}$$

Here, $w_m\ (j\omega)$ is the uncertainty bound on the sensitivity function which is given as $S(j\omega) = 1 - T\,(j\omega)$. Hence the tuning parameter has to be selected such that the resulting controller should satisfy the robust stability and robust performance constraints (eqs 47 & 52).

A. Selection of the tuning parameter (λ):

It is well-known that there is always a tradeoff in selecting the desired closed-loop tuning parameter (λ). Fast speed of response and good disturbance rejection are favored by choosing a small value of λ. However, stability and robustness are favored by a large value of λ. Hence, the choice of λ is entirely based on the experience of the operator with the control system. Based on many simulation studies, it is observed that the starting value of λ can be considered around the process time delay. If both nominal and robust control performances are achieved with this value, then this value for λ can be taken as the final value. If not, the value should be increased slightly till the nominal and robust control performances are achieved. To have clear guidelines for selection of λ, in the present work, systematic analysis is carried out by using maximum sensitivity (M_s) as the performance index. M_s is also a robust performance measure like Gain margin (GM) and Phase margin (PM) and is related to these margins as [23]

$$GM \geq M_s / (M_s - 1), \quad PM \geq 2\sin^{-1}(1/2M_s)$$

In the present work, the process time delay to process time constant ratio (θ/τ) is varied and corresponding M_s values are plotted. The λ value is chosen accordingly for required value of M_s. If the θ/τ ratio is varied further more we have to retune the controller for good set point tracking and robust performance.

IV. SIMULATION RESULTS

Simulation studies have been carried out on various UFOPTD and USOPTD processes and observed that the performance of the controllers obtained from Taylor's 1st and 2nd order approximations for the time delay provide oscillatory responses in all the cases. Hence, simulation results are provided here for other controller structures

Example-1:

An UFOPTD process with k_p= 1, τ = 1, and θ = 0.4 is considered here. For this process, the controller settings are calculated for the proposed method from eqs. 9,13,17. The tuning parameter is selected as λ = 1 and the controllers are obtained as K_c = 1.9341, τ_I = 4.9692, α_1 = 0.2667, α_2 = 0.0267, β_1 = 0.1349, β_2 = 0.0104 (Pade's ½ order), K_c = 1.9231, τ_I = 5.0, α = 0.2, β = 0.0769 (Pade's 1st order), K_c = 1.9349, τ_I = 4.9672, α_1 = 0.2, α_2 = 0.0133, β_1 = 0.0677, β_2 = 0.0052 (Pade's 2nd order). Set point weighting is considered as ε = 0.1. With these controller settings, the performances of the three approximations are compared by considering a unit step input at time t = 0 and a negative step disturbance of magnitude 0.1 at *t* =10 sec respectively. Figure 2 shows the responses for perfect model parameters. From the responses one can observe that all the three approximations provide almost same performance. Perturbations of +20% in K_P, θ and -20% in τ are considered and the corresponding responses are shown in Figure 3. Here Pade's 1st order shows oscillatory responses and Pade's 2nd order shows better performances than Pade's ½ order. It is also observed that for a higher perturbation in the process parameters, Pade's 2nd order is providing further better responses. For quantitative comparison, integral of absolute error (IAE) is considered and are given in Table 3. It can be observed from the IAE values that Pade's 2nd order method gives lesser IAE values.

To analyze the selection of λ for robustness, analysis has been carried out by selecting peak value of the sensitivity function (Ms). Figure 4 shows the variation of Ms for different values of θ/τ for various values of λ for Pade's ½ order. Figure 5 shows the variation of Ms for different values of θ/τ for various values of λ for Pade's first order and Figure 6 shows the variation of Ms for different values of θ/τ for various values of λ for Pade's 2nd order. It can be observed that from Figures 4-6 that, for lower values of θ/τ, Ms values are less indicating the stable response. For lower values of λ, Ms values are high indicating non-robust responses. Hence to obtain robust responses, selecting the tuning parameter as λ = 2 provides robust performances. Thus, it can be observed that one can go for higher values of λ to obtain robust control performances. To analyze the superiority of the methods, Ms values are plotted for different values of θ/τ with λ = 1 and are shown in Figure 7. It can be observed that 1st order and ½ order (graphs coinciding) approximation shows higher values of Ms and hence is not recommended. As 2nd order approximation shows lesser values, this is recommended.

For comparison with previous method, method of Shamsuzzoha and Lee [24] is considered and designed the controllers. Pade's 2nd order method is considered for the proposed method because it provides improved performances than that of other approximations. To have the same closed loop tuning parameter, λ = 1 is considered for both the methods. The controller parameters obtained for Shamsuzzoha and Lee [24] are K_c = 0.0354, τ_I = 0.2667, τ_D = 0.1, a = 10.9346, b = 0.2986, α = 10.9346. Figure 8 shows the closed loop responses for a unit step input at time t = 0 and a negative step disturbance of magnitude 0.1 at *t* =15 sec respectively for perfect model parameters. The proposed method shows improved performances. To evaluate the closed-loop performance in the presence of the plant model mismatch, perturbations of +20% in K_P ,+10% in θ and -10% in τ are considered and the corresponding responses are shown in Figure 9. Here also, the proposed method gives improved performances. For quantitative comparison, IAE values are considered and are given in Table 4. It can be observed that the proposed method (with Pade's 2nd order approximation) gives lesser IAE values when compared with that of Shamsuzzoha and Lee [24] method, who have already shown the advantage of their method over many methods.

Example-2:

Here, an USOPTD process with two unstable poles is (eq. 26) considered where $K_P = 2$, $a_1 = 3$, $a_2 = -4$ and $\theta = 0.3$. The tuning parameter is selected as $\lambda = 1.5$ and the controller parameters are obtained as $K_c = 0.4367$, $\tau_I = 2.2379$, $\tau_D = 7.9787$, $\alpha_1 = 0.2$, $\alpha_2 = 0.015$, $\beta_1 = 0.1098$, $\beta_2 = 0.0066$ (Pade's ½ order), $K_c = 0.4289$, $\tau_I = 2.2163$, $\tau_D = 8.087$, $\alpha = 0.15$, $\beta = 0.0653$ (Pade's 1st order), $K_c = 0.4372$, $\tau_I = 2.239$, $\tau_D = 7.9732$, $\alpha_1 = 0.15$, $\alpha_2 = 0.0075$, $\beta_1 = 0.0594$, $\beta_2 = 0.0033$ (Pade's 2nd order). Set point weighting is considered as $\varepsilon = 0.1$. With these controller settings, the performances of the three approximations are compared by considering a unit step input at time t = 0 and a negative step disturbance of magnitude 0.1 at t =30 sec respectively. Figure 10 shows the responses for perfect model parameters. From the responses one can observe that all the three approximations provide almost same performance. Perturbations of +25% in K_P, +10% in θ and -20% in τ_1, τ_2 are considered and the corresponding responses are shown in Figure 11. Here also Pade's 2nd order is gives better performances. It is also observed that for higher perturbations in the process parameters, Pade's 2nd order gives better performance. Figure 12 shows the variation of Ms for different values of θ/τ with $\lambda = 1.5$. It can be observed that 1st order approximation shows higher values of Ms and hence is not recommended. Pade's ½ order approximation shows higher values when compared to that of 2nd order and hence 2nd order approximation for the time delay term is recommended. To analyze the robustness further, robust stability condition is verified for uncertainty in time delay. Figure 13 shows the magnitude of complementary sensitivity function for uncertainties in time delay from 10% - 50%. It can be observed that the robust stability condition is satisfied according to eq. 49.

Example-3:

Here, an USOPTD process (eq. 25) with one unstable pole and one stable pole is used where $K_P = 1$, $\tau_1 = 2.07$, $\tau_2 = 5$, and $\theta = 0.932$. The parameters obtained after converting to eq. 28 are $K_P = 1$, $a_1 = -10.35$, $a_2 = -2.93$ and $\theta = 0.932$. The tuning parameter is selected as $\lambda = 1.5$ and the controller parameters are obtained as $K_c = 6.4564$, $\tau_I = 6.4358$, $\tau_D = 1.4130$, $\alpha_1 = 0.6260$, $\alpha_2 = 0.1470$, $\beta_1 = 0.2873$, $\beta_2 = 0.0481$ (Pade's ½ order), $K_c = 6.4285$, $\tau_I = 6.4409$, $\tau_D = 1.4135$, $\alpha = 0.4695$, $\beta = 0.1528$ (Pade's 1st order), $K_c = 6.4572$, $\tau_I = 6.4357$, $\tau_D = 1.4130$, $\alpha_1 = 0.4695$, $\alpha_2 = 0.0735$, $\beta_1 = 0.1301$, $\beta_2 = 0.0240$ (Pade's 2nd order). The set point weighting is considered as $\varepsilon = 0.1$. With these controller settings, the performances of the three approximations are compared by considering a unit step input at time t = 0 and a negative step disturbance of magnitude 0.1 at *t* =30 sec respectively. Figure 14 shows the responses for perfect model parameters. From the responses one can observe that all the three approximations provide almost same performance. Perturbations of +20% in K_P, +10% in θ, -20% in τ_1, -10% in τ_2 are considered and the corresponding responses are shown in Figure 15. Here Pade's 1st order is going oscillator and Pade's 2nd order is giving better performances than Pade's ½ order. It can be observed that for higher perturbations in the process parameters, Pade's 2nd order gives better response. For quantitative comparison, the IAE values are considered and are given in Table 5. It can be observed that Pade's 2nd order method gives lesser IAE values.

For comparison with previous methods, Tan et al. [13] method is considered. For the proposed method Pade's 2nd order is considered and compared with Tan et al [13] method by giving a unit step change in the set point and a unit negative step input in the load disturbance at t = 35 sec respectively. Figure 16 shows the responses for perfect model parameters. From the responses one can observe that the proposed method (Pade's 2nd order approximation) gives better performance. Perturbations of +10% in K_P, θ and -10% in τ_1, τ_2 are considered and the corresponding responses are shown in Figure 17. For quantitative comparison, the IAE values are considered and are given in Table 6. It can be observed that Pade's 2nd order method gives lesser IAE values than that of Tan et al [13].

Example-4:

An USOPTD process with an integrator (eq. 27) is considered here. The process is given as

$$G_p(s) = \frac{e^{-0.2s}}{s(s-1)}$$

However, to use the proposed method, the process is considered for convince as

$$G_p(s) = \frac{e^{-0.2s}}{(s-0.01)(s-1)}$$

This in turn can be written as

$$G_p(s) = \frac{100e^{-0.2s}}{100s^2 - 101s + 1}$$

Where K_P = 100, a_1 = 100, a_2 = -101 and θ = 0.2. The tuning parameter is selected as λ = 1and the controller parameters are obtained as K_c = 1.6274, τ_I = 3.1805, τ_D =1.7579, α_1 = 0.1333, α_2 = 0.0067, β_1 = 0.0819, β_2 = 0.0034 (Pade's ½ order), K_c = 1.622, τ_I = 3.1804, τ_D = 1.76, α = 0.1, β = 0.051 (Pade's 1st order), K_c = 1.6276, τ_I = 3.1805, τ_D =1.7578, α_1 = 0.1, α_2 = 0.0033, β_1 = 0.0485, β_2 = 0.0017 (Pade's 2nd order). The set point weighting is considered as ε = 0.1. With these controller settings, the performances of the three approximations are compared by considering a unit step input at time t = 0 and a negative step disturbance of magnitude 0.1 at t = 30 sec respectively. Figure 18 shows the responses for perfect model parameters. From the responses one can observe that all the three approximations provide almost same performance. Perturbations of +30% in K_P, θ, -30% in τ are considered and the corresponding responses are shown in Figure 19. Here also Pade's 2nd order provides better performances. For quantitative comparison, the IAE values are considered and are given in Table 7. It can be observed that Pade's 2nd order method gives lesser IAE values. For further analysis of robustness, robust stability is evaluated by using complementary sensitivity function for uncertainty of +50% in the time delay and is given in Figure 20. It can be observed that λ = 0.5 does not follow the robust stability condition where as λ = 1 and λ = 1.5 satisfy the robust stability condition (eq. 49) and selecting λ = 1.5 gives more robust control performances. In fact, it is true that robustness of the closed loop increases as the value of the tuning parameter is increased.

V. CONCLUSIONS

Different approximations for time delay terms are considered to derive the controller parameters in direct synthesis method for UFOPTD and USOPTD processes. Of all the approximations, Pade's 2nd order approximation provides good nominal and robust closed loop performances. Systematic analysis has been carried out for selection of the tuning parameter using maximum sensitivity. Pade's 2nd order approximation provides lower Ms values compared to other approximations. The proposed design using Pade's 2nd order gives improved closed loop performances when compared to that of the recently reported methods in the literature and also provides low IAE values for nominal as well as perturbed conditions.

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